

Vertical emittance reduction via coupling resonance driving terms correction: theory and experimental results at the ESRF

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**XVIIIth European Synchrotron Radiation Light Source Workshop 2010
ELETTRA, Trieste, 25-26 November 2010**

Outlines

- Vertical emittance s in presence of coupling
- Coupling correction via Resonance Driving Terms
- Application to the ESRF storage ring
- Preserving small vertical emittance during beam delivery
- Towards ultra-small vertical emittance

Vertical emittance in absence of coupling

Eigen-emittance:

$$\mathcal{E}_u = \frac{1}{2} \frac{\oint \{\mathcal{H}_x^2(s) d(s)\} ds}{\oint \{b_{RF}(s) - D_x(s) b_{\delta x}(s)\} ds}$$

$$\mathcal{E}_v = \frac{1}{2} \frac{\oint \{\mathcal{H}_y^2(s) d(s)\} ds}{\oint \{b_{RF}(s) - D_y(s) b_{\delta y}(s)\} ds}$$

RMS emittance:

$$\epsilon_r = \sqrt{\sigma_r(s) \sigma_p(s) - \sigma_{rp}^2(s)}$$

Measurable emittance:

$$\mathbb{E}_r = \frac{\sigma_r^2(s)}{\beta_r(s)} = \frac{\langle r^2(s) \rangle - (\delta D_r(s))^2}{\beta_r(s)}$$

The three definitions are equivalent and

$E_y = \mathcal{E}_v = \mathcal{E}_y = \text{const.}$ If vert. disp. $D_y = 0$, $E_y = \mathcal{E}_v = \mathcal{E}_y \approx 0$

Vertical emittances in presence of coupling

Constant eigen-emittance [See B. Nash *et al.* PRSTAB 9, 032801, 2006]:

$$\mathcal{E}_v = \frac{1}{2} \frac{\oint d(s) \{ C^2 \mathcal{H}_y^2(s) + [S_-^2 + S_+^2] \mathcal{H}_x^2(s) \} ds}{\oint \{ b_{RF}(s) - C^2 D_y(s) b_{\delta y}(s) - [S_-^2 - S_+^2] D_x(s) b_{\delta x}(s) \} ds}$$

RMS **projected s-dependent** emittance:

$$\epsilon_y = \sqrt{\sigma_y \sigma_p - \sigma_{yp}^2} = \sqrt{(C^2 \mathcal{E}_v + [S_-^2 + S_+^2] \mathcal{E}_u)^2 - (2S_+ S_- \mathcal{E}_u)^2}$$

Measurable **apparent s-dependent** emittance:

$$\mathbb{E}_y = \frac{\sigma_y^2}{\beta_y} = C^2 \mathcal{E}_v + [S_-^2 + S_+^2 - 2S_- S_+ \cos(q_+ - q_-)] \mathcal{E}_u$$

In **absence** of coupling $C=1$, $S_- = S_+ = 0$ and $\mathbb{E}_y = \mathcal{E}_v = \epsilon_y = \text{const}$

Vertical emittances in presence of coupling

- Coupling sources (tilted quads, misaligned sextupoles, ID error fields, etc.) generate skew quad fields $\mathbf{J}_1(s)$
- $\mathbf{J}_1(s)$ generate two *Resonance Driving Terms* (RDTs) $f(s)$

$$f_a = M_{ab}(\beta, \varphi) \mathbf{J}_{b,1}$$

Linear dependence!!

$$\mathcal{C} = \cosh(2\mathcal{P}),$$

$$\mathcal{S}_- = \frac{\sinh(2\mathcal{P})}{\mathcal{P}} |f_{1001}|,$$

$$\mathcal{S}_+ = \frac{\sinh(2\mathcal{P})}{\mathcal{P}} |f_{1010}|,$$

$$\mathcal{P} = \sqrt{-|f_{1001}|^2 + |f_{1010}|^2},$$

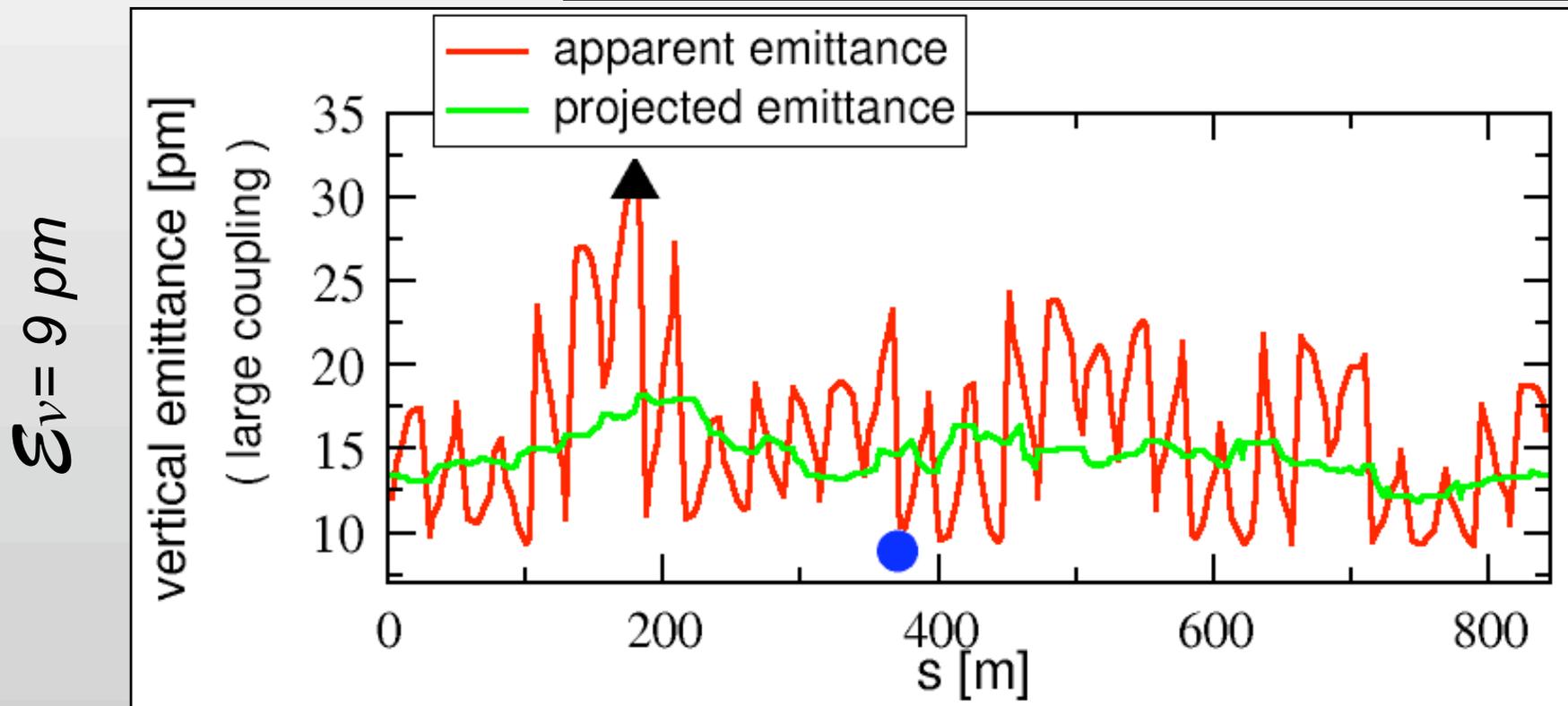
$$f_{\begin{smallmatrix} 1001 \\ 1010 \end{smallmatrix}} = \frac{\sum_w^W J_{w,1} \sqrt{\beta_x^w \beta_y^w} e^{i(\Delta\phi_{w,x} \mp \Delta\phi_{w,y})}}{4(1 - e^{2\pi i(Q_u \mp Q_v)})}$$

$$q_- = \arg\{f_{1001}\}, \quad q_+ = \arg\{f_{1010}\}$$

Vertical emittances in **presence** of coupling

Apparent emittance: $\mathbb{E}_y = \mathcal{C}^2 \mathcal{E}_v + [\mathcal{S}_-^2 + \mathcal{S}_+^2 - 2\mathcal{S}_-\mathcal{S}_+ \cos(q_+ - q_-)] \mathcal{E}_u$

Projected emittance: $\epsilon_y = \sqrt{(\mathcal{C}^2 \mathcal{E}_v + [\mathcal{S}_-^2 + \mathcal{S}_+^2] \mathcal{E}_u)^2 - (2\mathcal{S}_+\mathcal{S}_-\mathcal{E}_u)^2}$



Vertical emittances in presence of coupling

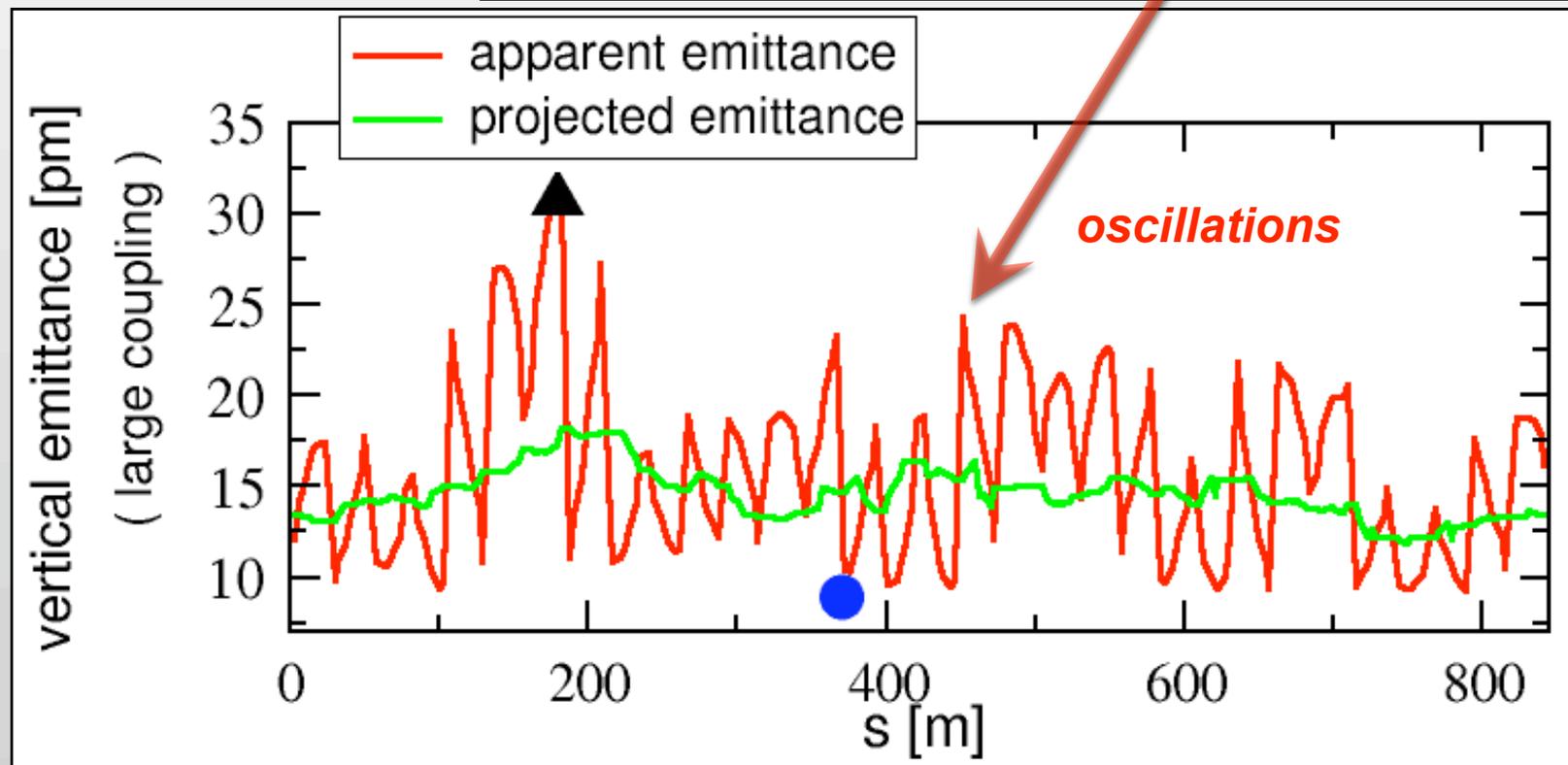
Apparent emittance:

$$\mathbb{E}_y = \mathcal{C}^2 \mathcal{E}_v + [\mathcal{S}_-^2 + \mathcal{S}_+^2 - 2\mathcal{S}_- \mathcal{S}_+ \cos(q_+ - q_-)] \mathcal{E}_u$$

Projected emittance:

$$\epsilon_y = \sqrt{(\mathcal{C}^2 \mathcal{E}_v + [\mathcal{S}_-^2 + \mathcal{S}_+^2] \mathcal{E}_u)^2 - (2\mathcal{S}_+ \mathcal{S}_- \mathcal{E}_u)^2}$$

$\mathcal{E}_v = 9 \text{ pm}$

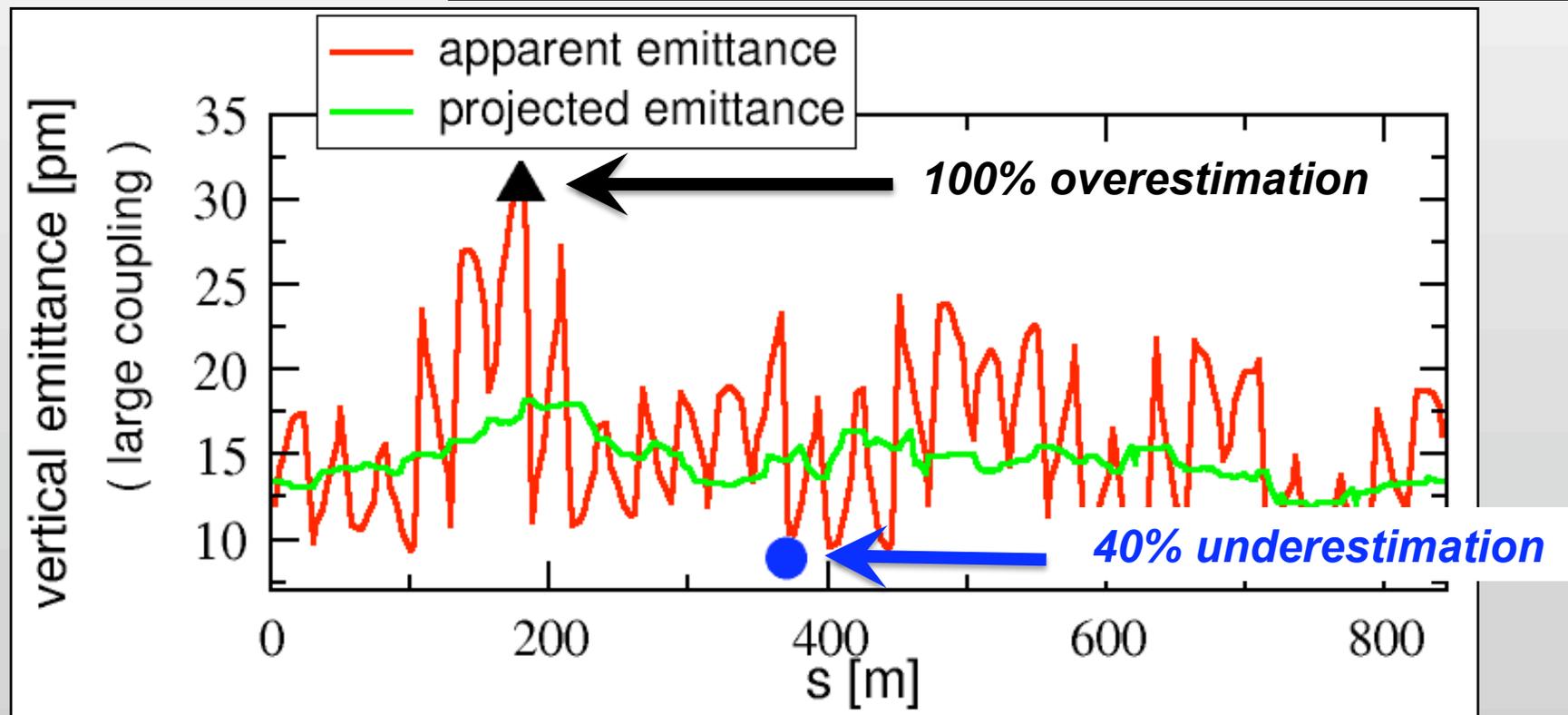


Vertical emittances in presence of coupling

Apparent emittance: $\mathbb{E}_y = C^2 \mathcal{E}_v + [S_-^2 + S_+^2 - 2S_- S_+ \cos(q_+ - q_-)] \mathcal{E}_u$

Projected emittance: $\epsilon_y = \sqrt{(C^2 \mathcal{E}_v + [S_-^2 + S_+^2] \mathcal{E}_u)^2 - (2S_+ S_- \mathcal{E}_u)^2}$

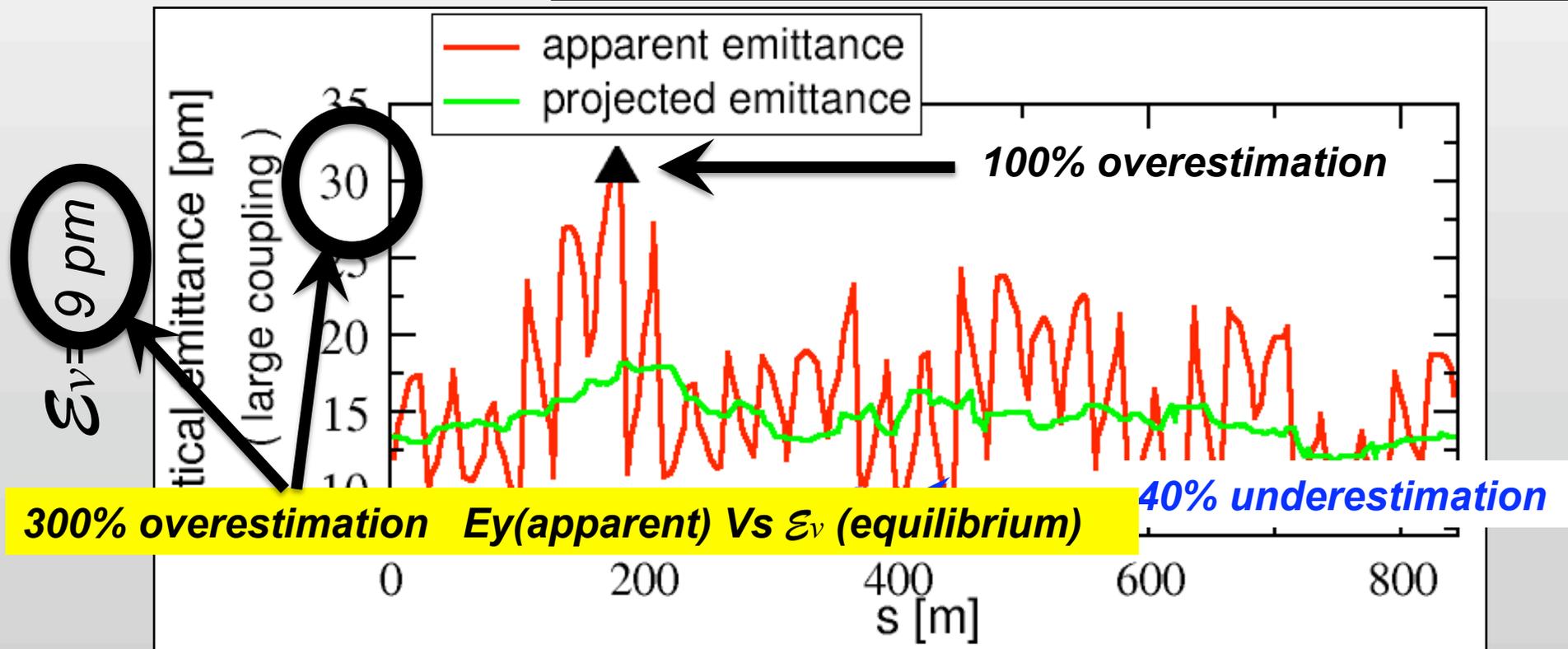
$\mathcal{E}_v = 9 \text{ pm}$



Vertical emittances in presence of coupling

Apparent emittance: $\mathbb{E}_y = C^2 \mathcal{E}_v + [S_-^2 + S_+^2 - 2S_- S_+ \cos(q_+ - q_-)] \mathcal{E}_u$

Projected emittance: $\epsilon_y = \sqrt{(C^2 \mathcal{E}_v + [S_-^2 + S_+^2] \mathcal{E}_u)^2 - (2S_+ S_- \mathcal{E}_u)^2}$



Vertical emittances in presence of coupling

Apparent emittance:

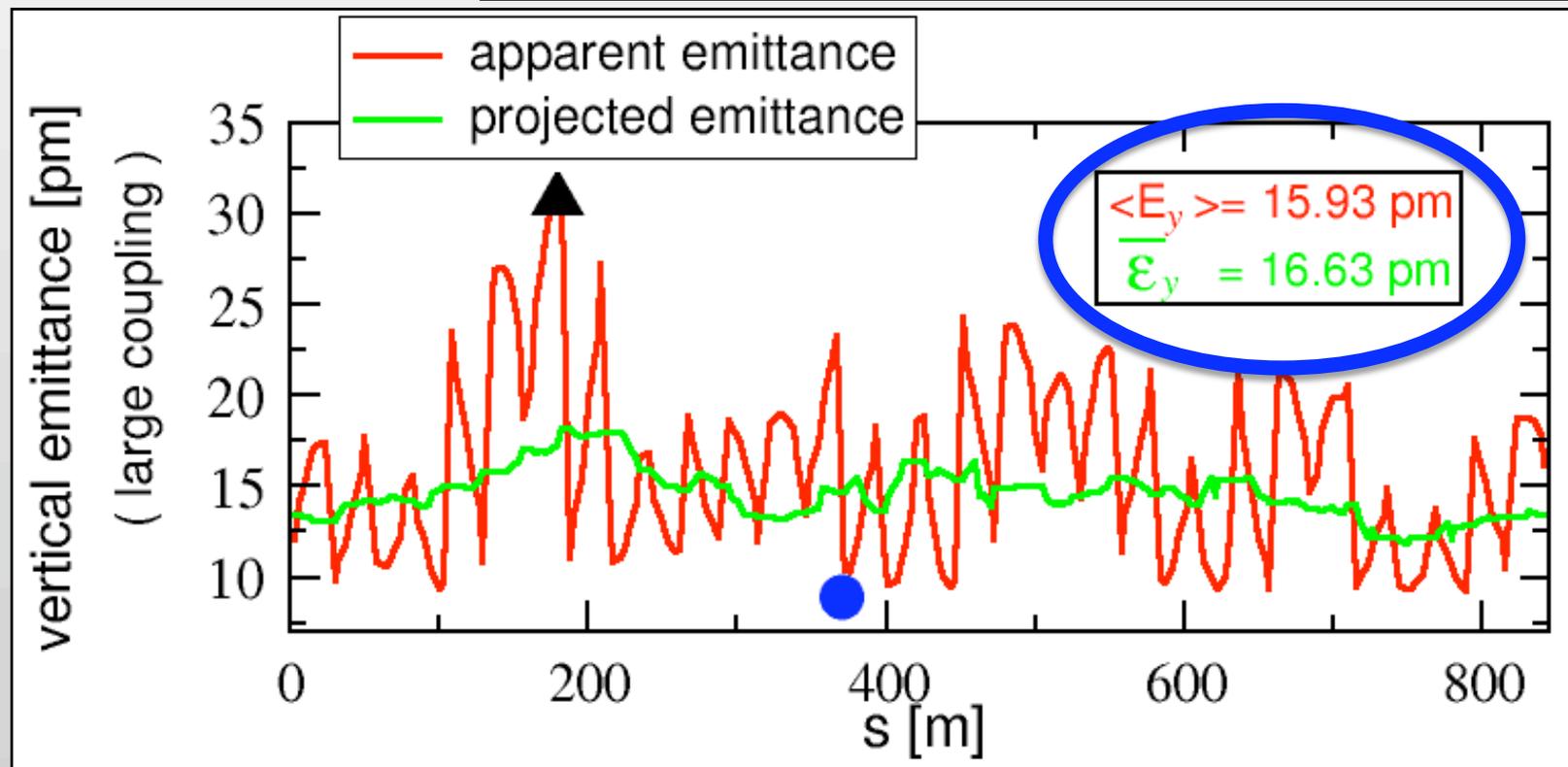
$$\langle E_y \rangle = C^2 \mathcal{E}_v + [S_-^2 + S_+^2 - 2S_- S_+ \cos(q_+ - q_-)] \mathcal{E}_u$$

Average over the ring

Projected emittance:

$$\langle \epsilon_y \rangle = \sqrt{(C^2 \mathcal{E}_v + [S_-^2 + S_+^2] \mathcal{E}_u)^2 - (2S_+ S_- \mathcal{E}_u)^2}$$

$\mathcal{E}_v = 9 \text{ pm}$



Coupling correction via Resonance Driving Terms

The lower the vertical dispersion and the coupling RDTs, the smaller the vertical emittances

- Vertical dispersion D_y is linear in the skew quad strengths J_1 : once measured may be corrected via SVD of its response matrix
- Coupling RDTs are linear in J_1 too, and a response matrix may be easily defined and used for correction (fast, direct, no iteration needed)

Procedure [already independently developed by R. Tomas (for ALBA)]:

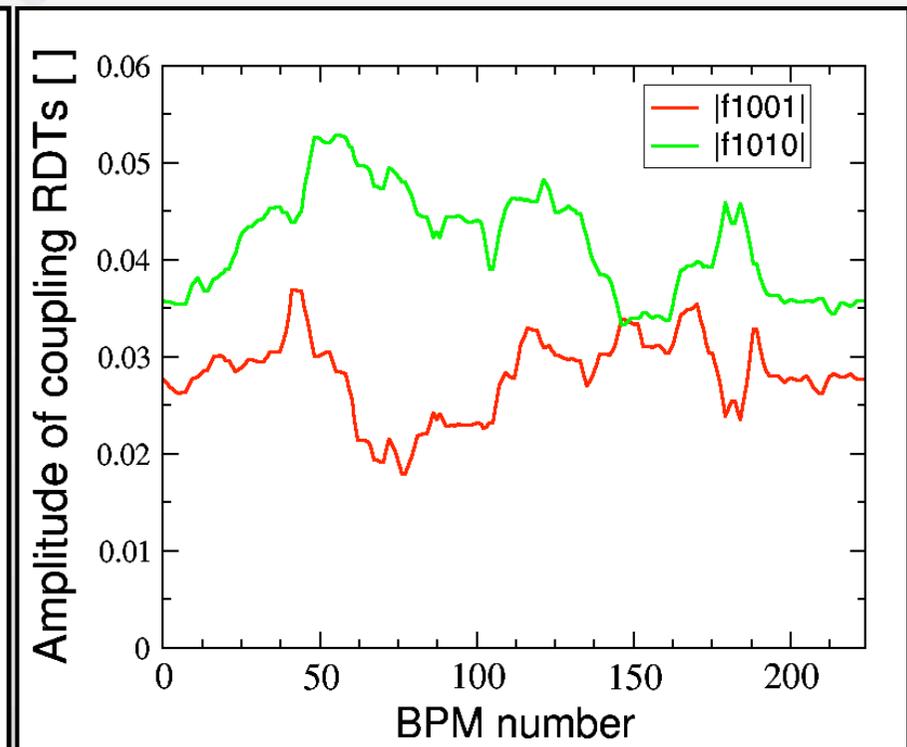
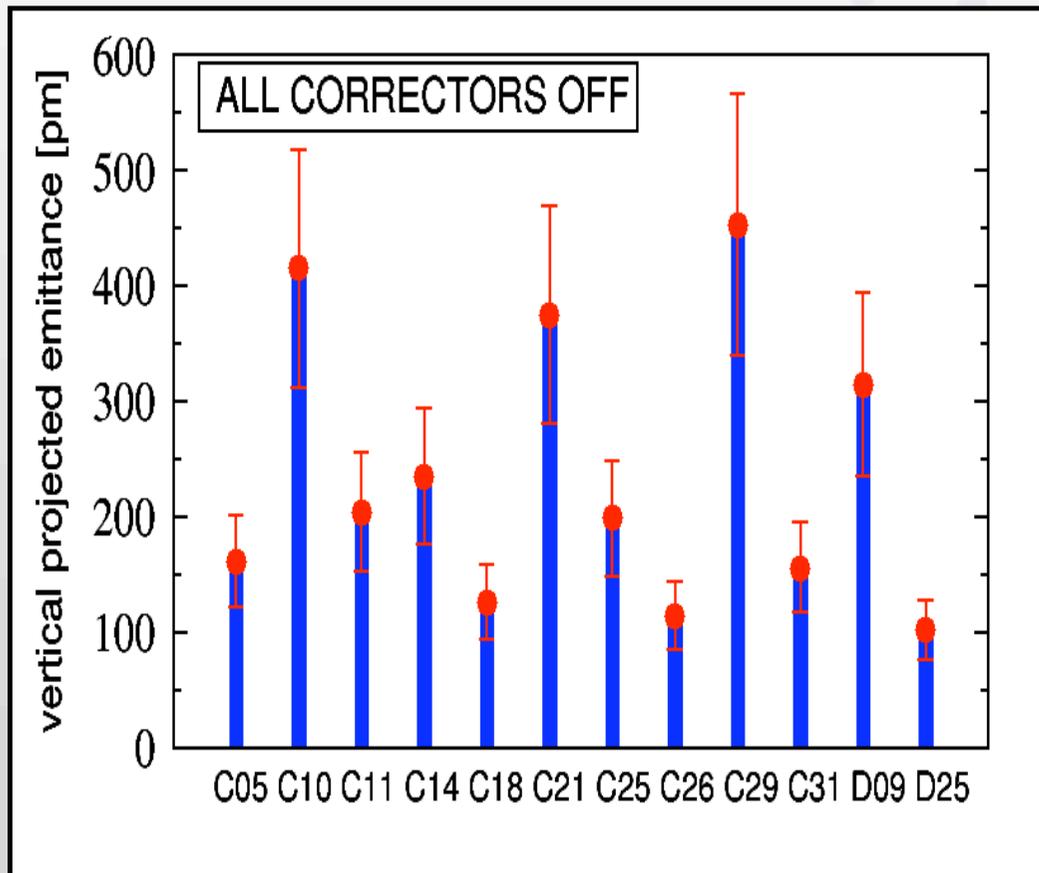
1. Define an error lattice model (quad tilts, etc. from ORM or TbT BPM data) => RDTs and D_y \vec{F}
2. Evaluate response matrix of the available skew correctors M
3. Find via SVD a corrector setting \vec{J} that minimizes both RDTs and D_y

$$\vec{J} = -M \vec{F} \quad \text{to be pseudo-inverted}$$

Application to the ESRF storage ring

First RDT correction: January 16th 2010

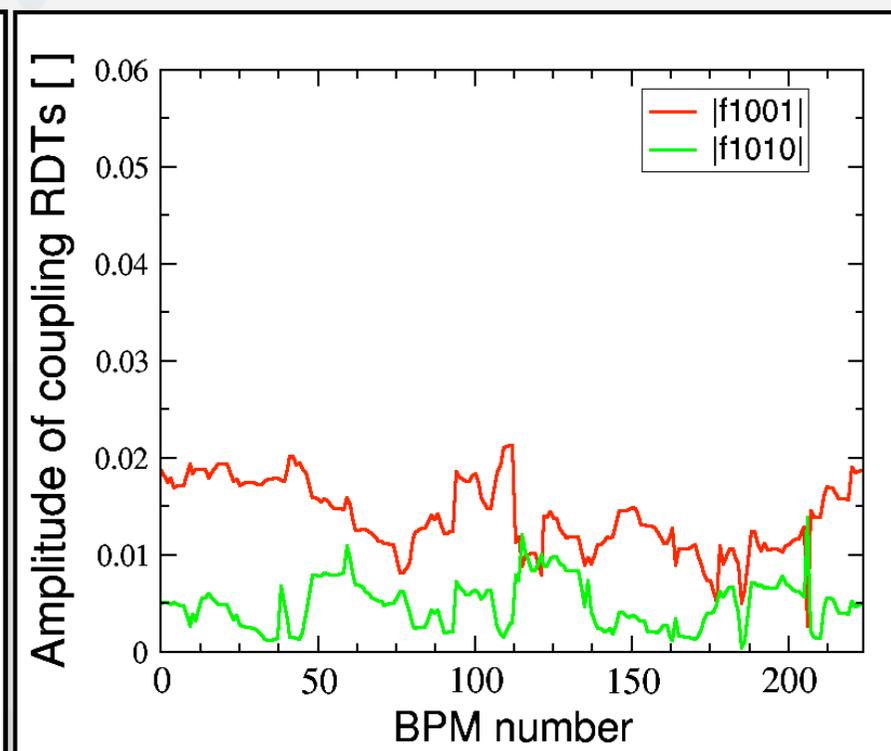
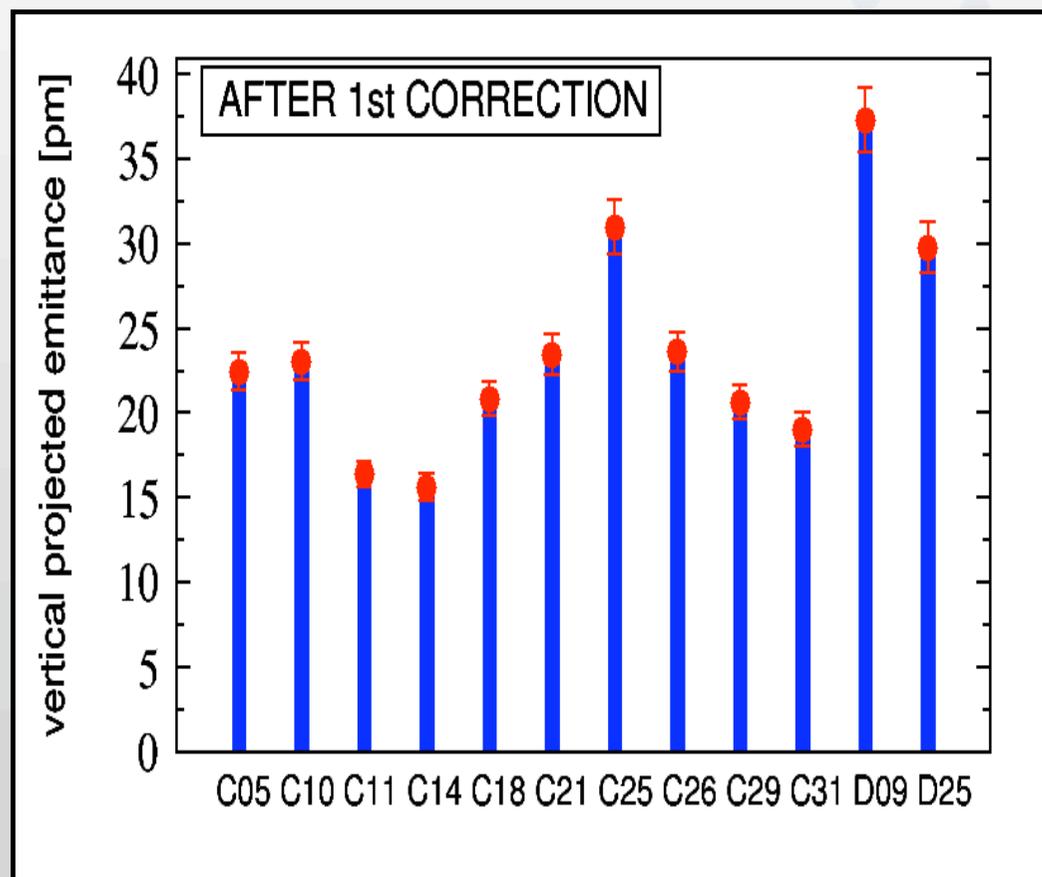
All skew correctors OFF: $\bar{\epsilon}_y \pm \delta\epsilon_y = 237 \pm 122 \text{ pm}$



Application to the ESRF storage ring

First RDT correction: January 16th 2010

1st ORM measur. and RDT correction: $\bar{\epsilon}_y \pm \delta\epsilon_y = 23.6 \pm 6.3 \text{ pm}$

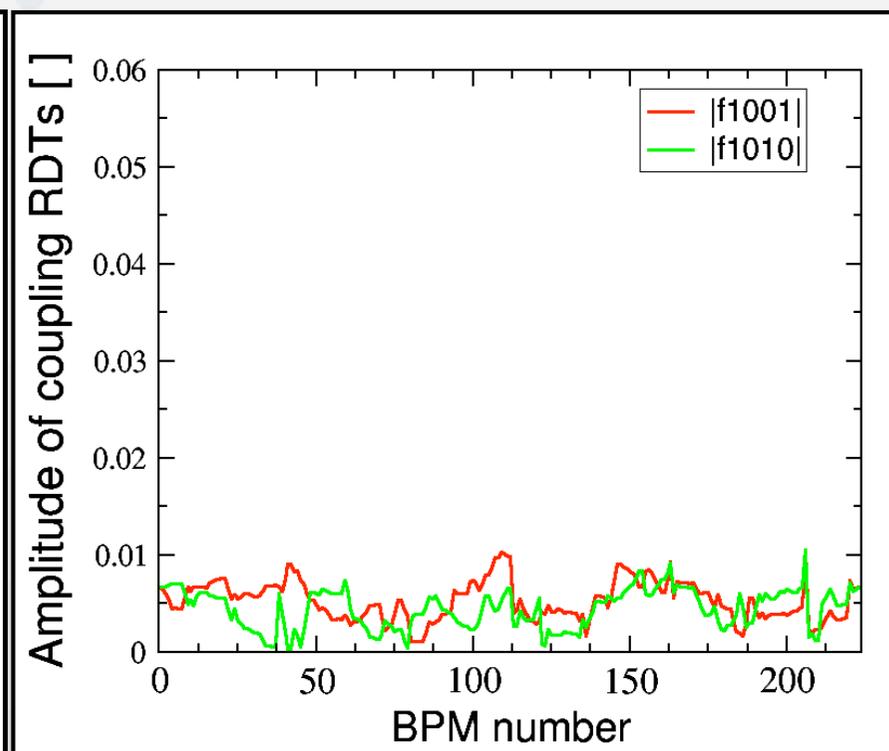
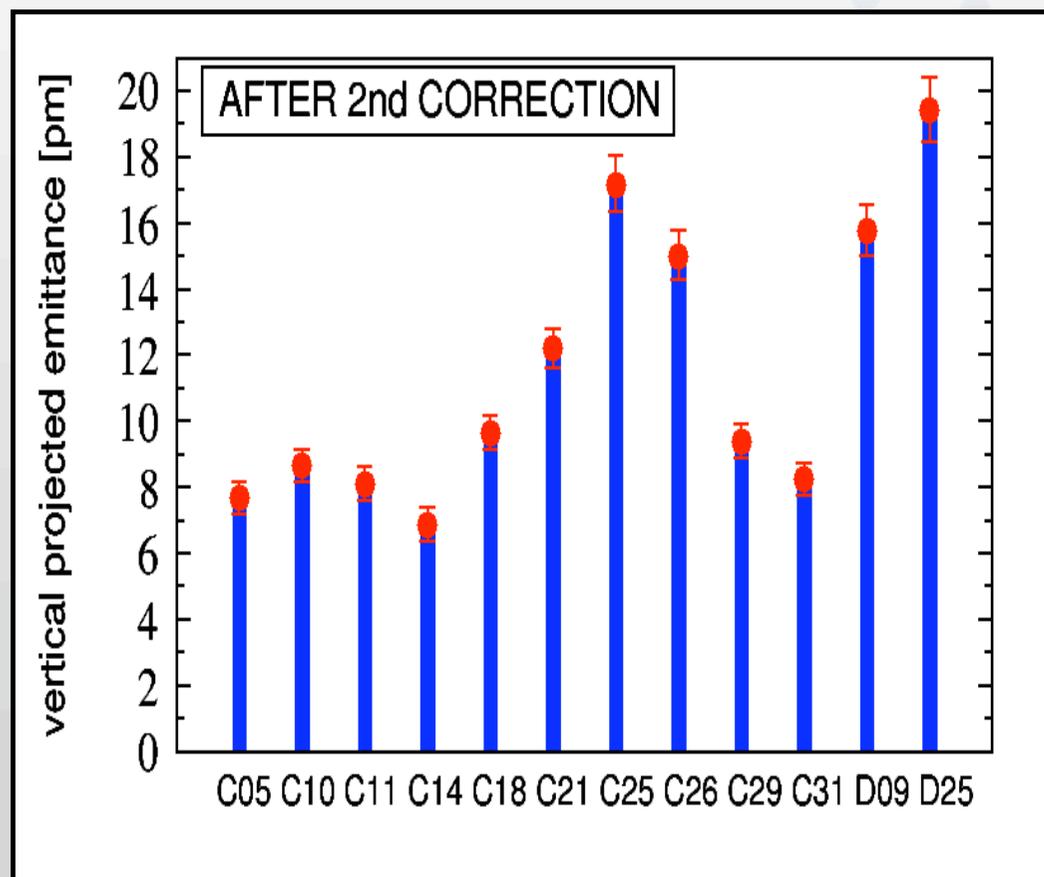


~20 min. for ORM
a few seconds for RDT correction

Application to the ESRF storage ring

First RDT correction: January 16th 2010

2nd ORM measur. and RDT correction: $\bar{\epsilon}_y \pm \delta\epsilon_y = 11.5 \pm 4.3 \text{ pm}$

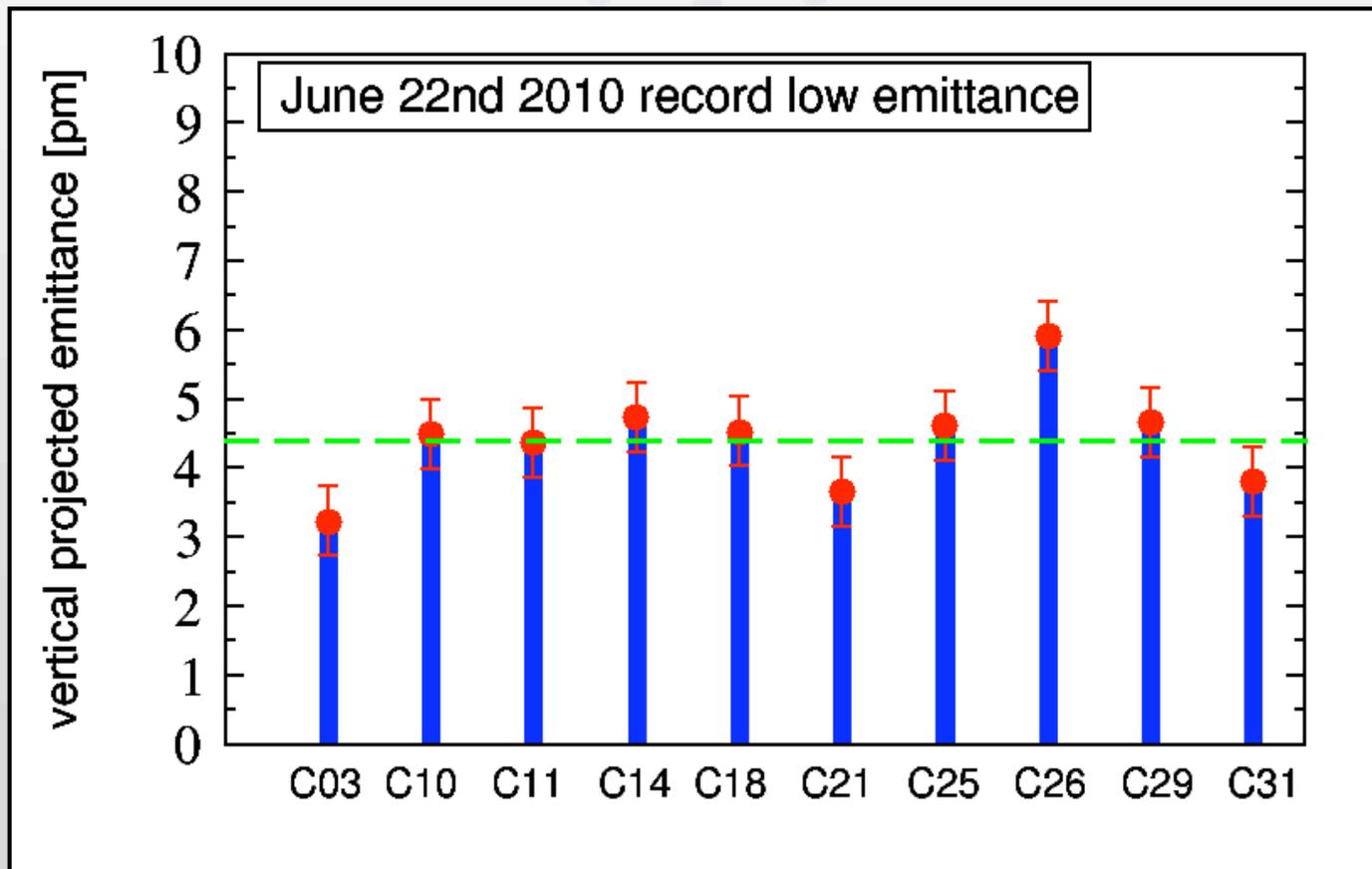


~20 min. for ORM
a few seconds for RDT correction

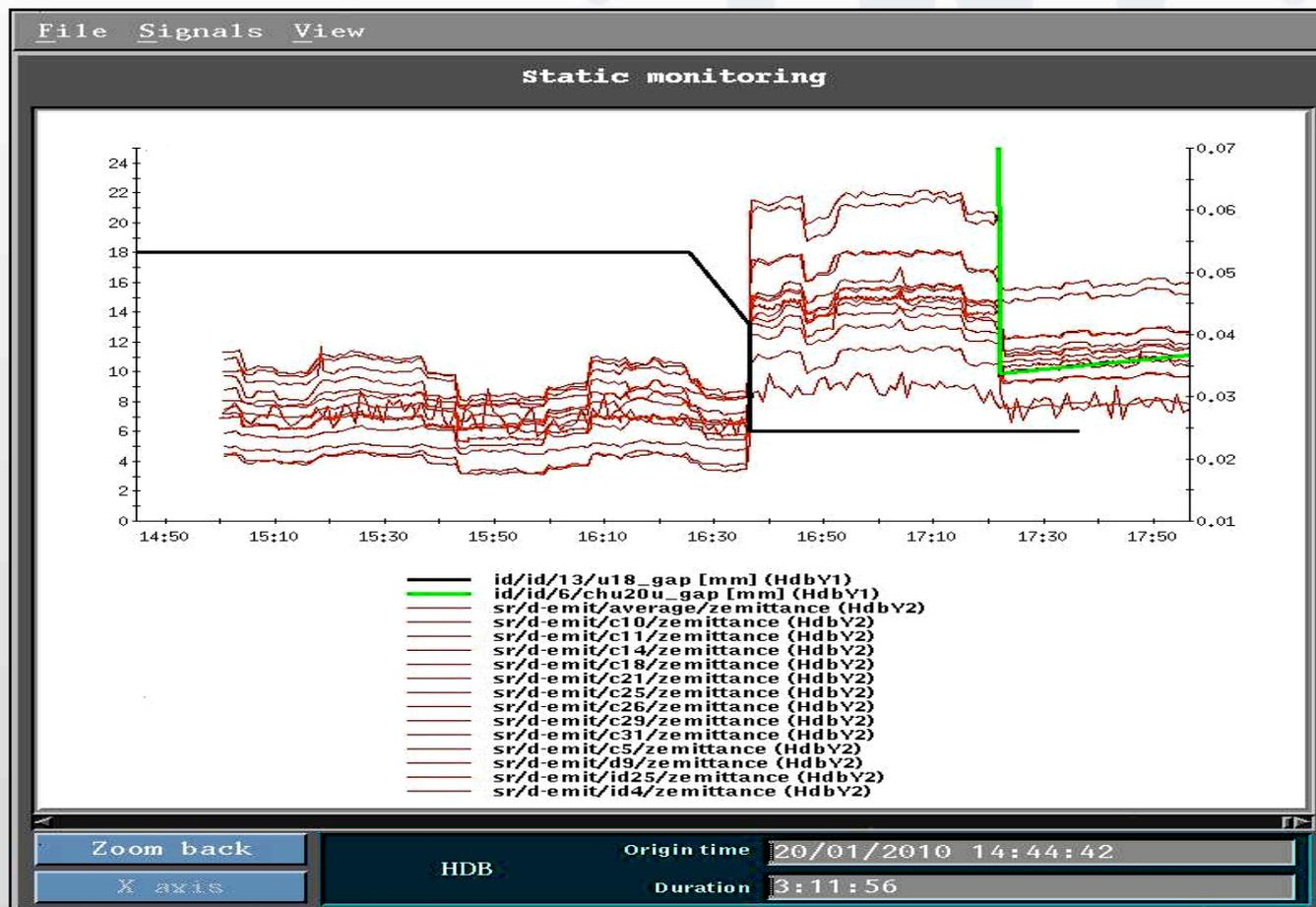
Application to the ESRF storage ring

ESRF record-low vertical emittance: June 22nd 2010

At ID gaps open: $\bar{\epsilon}_y \pm \delta\epsilon_y = 4.4 \pm 0.7$ pm



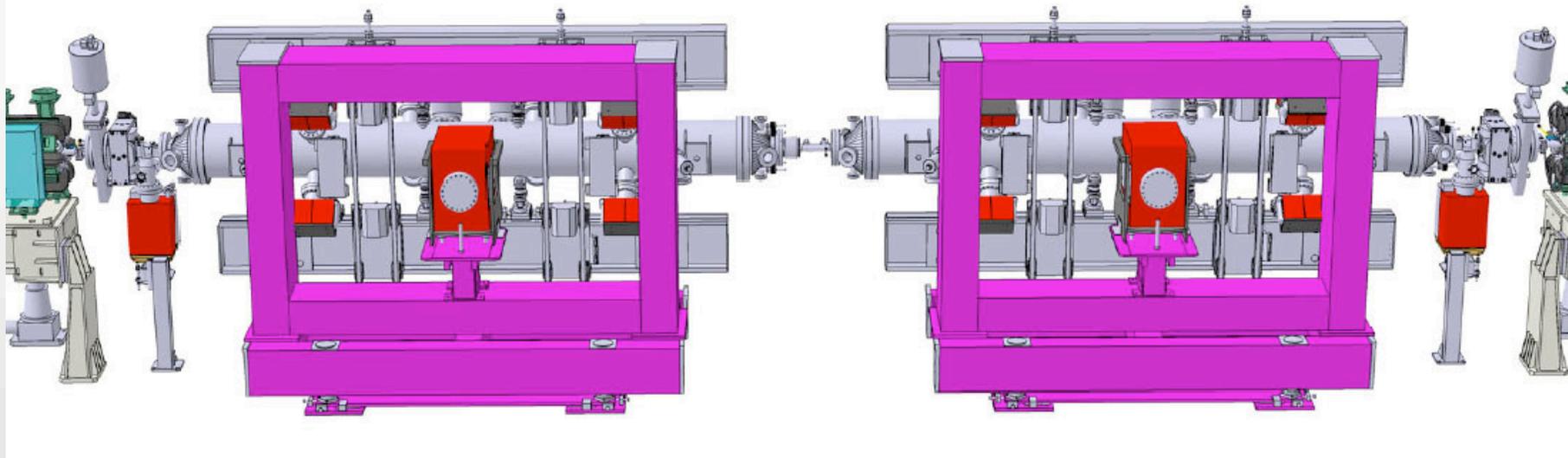
Preserving small vertical emittance during beam delivery



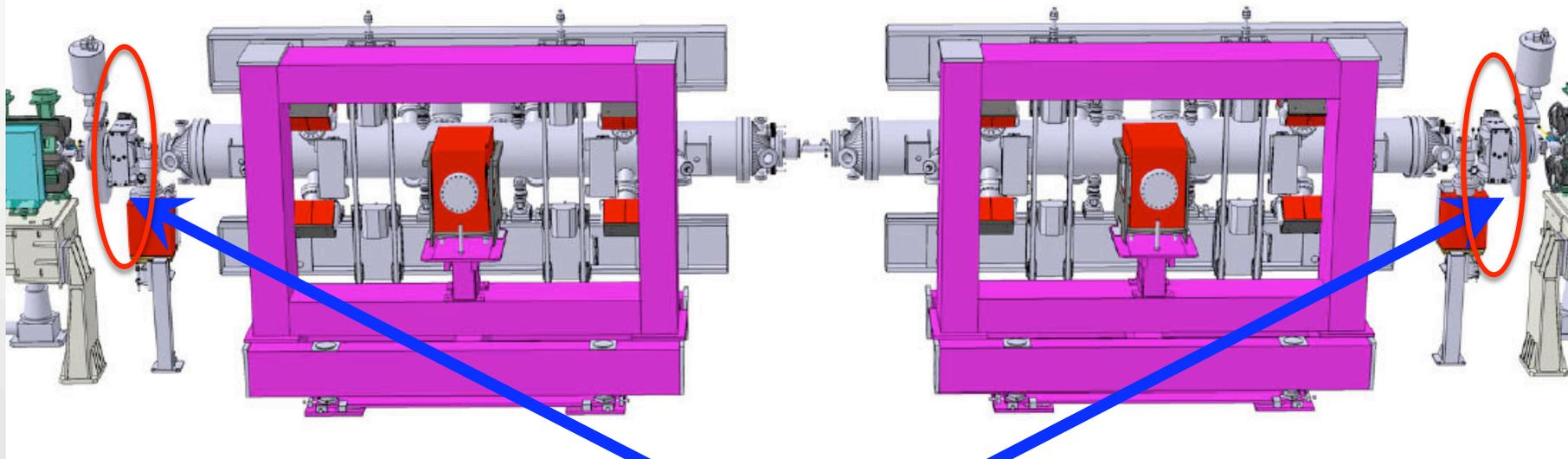
- Low coupling may not be preserved during beam delivery because of continuous changes of ID gaps that vary coupling along the ring

Apparent emittance measured at 14 monitors on Jan. 20th 2010, during beam delivery and movements of two ID gaps movements

Preserving small vertical emittance during beam delivery

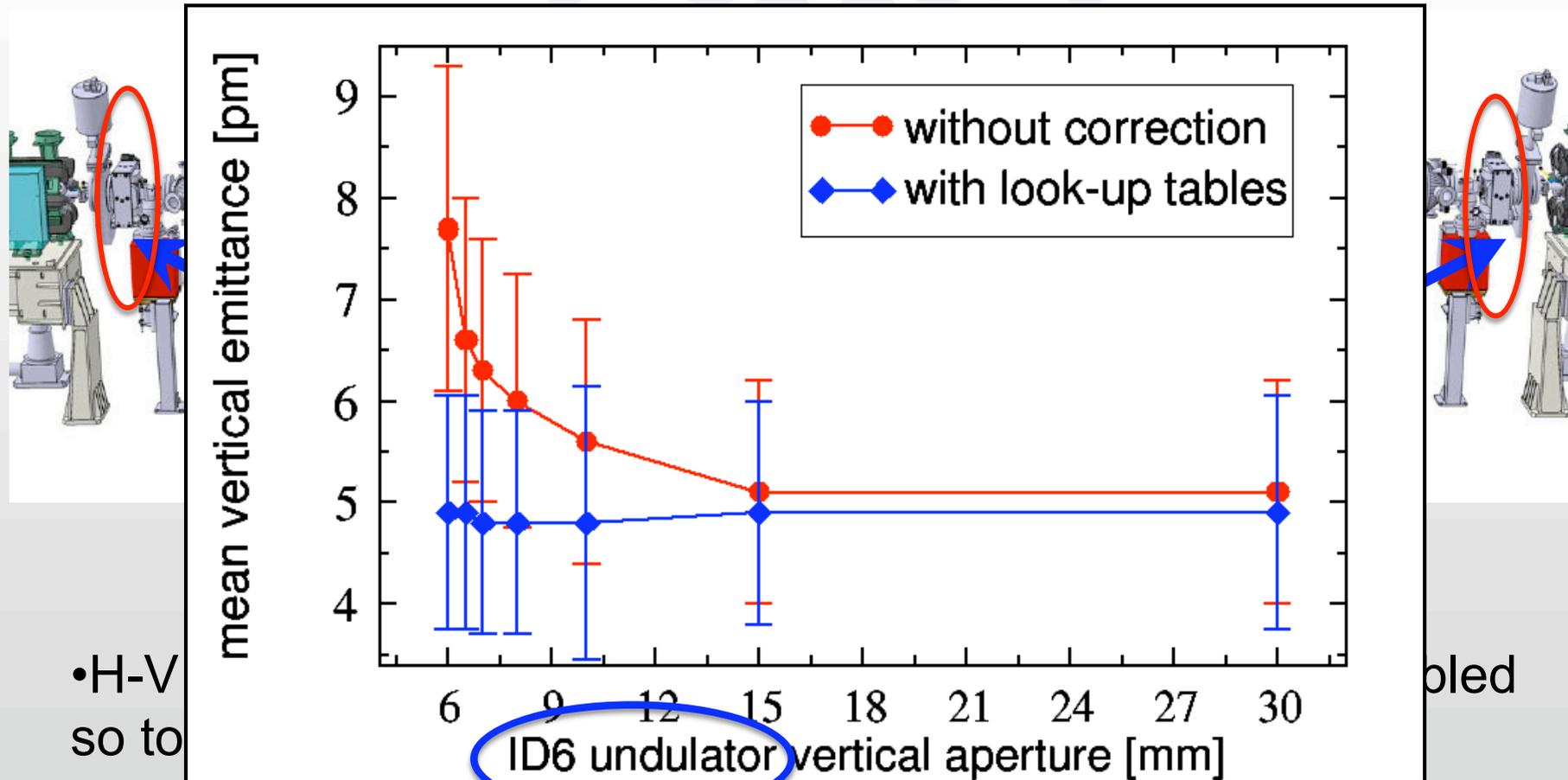


Preserving small vertical emittance during beam delivery



- H-V steerers at the ends of an ID straight section were cabled so to provide skew quad fields
- Look-up tables (corrector currents Vs ID gap aperture) were defined so to preserve the vertical emittance at any gap value.

Preserving small vertical emittance during beam delivery

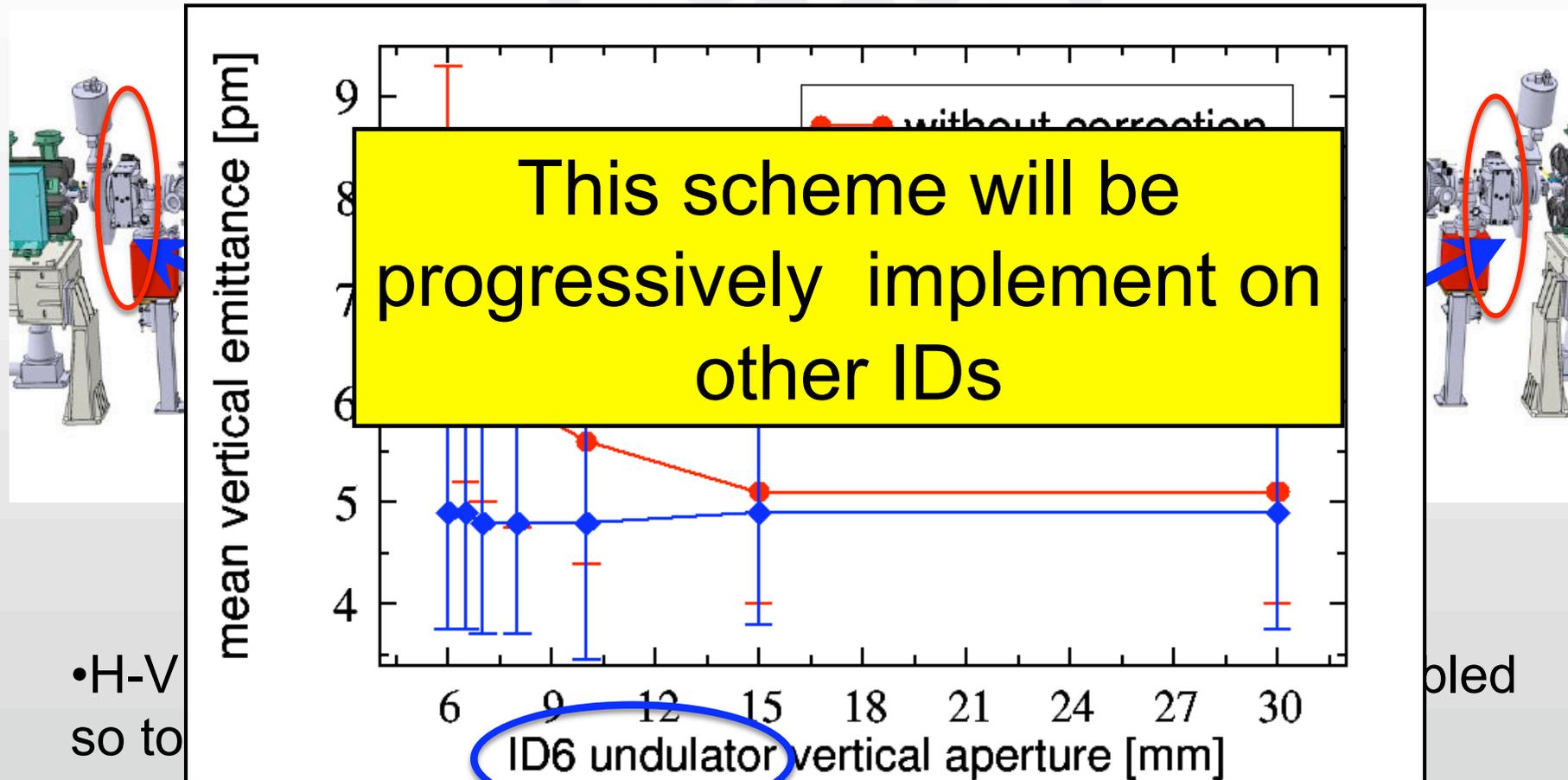


•H-V
so to

oled

•Look-up tables (corrector currents Vs ID gap aperture) were defined so to preserve the vertical emittance at any gap value.

Preserving small vertical emittance during beam delivery



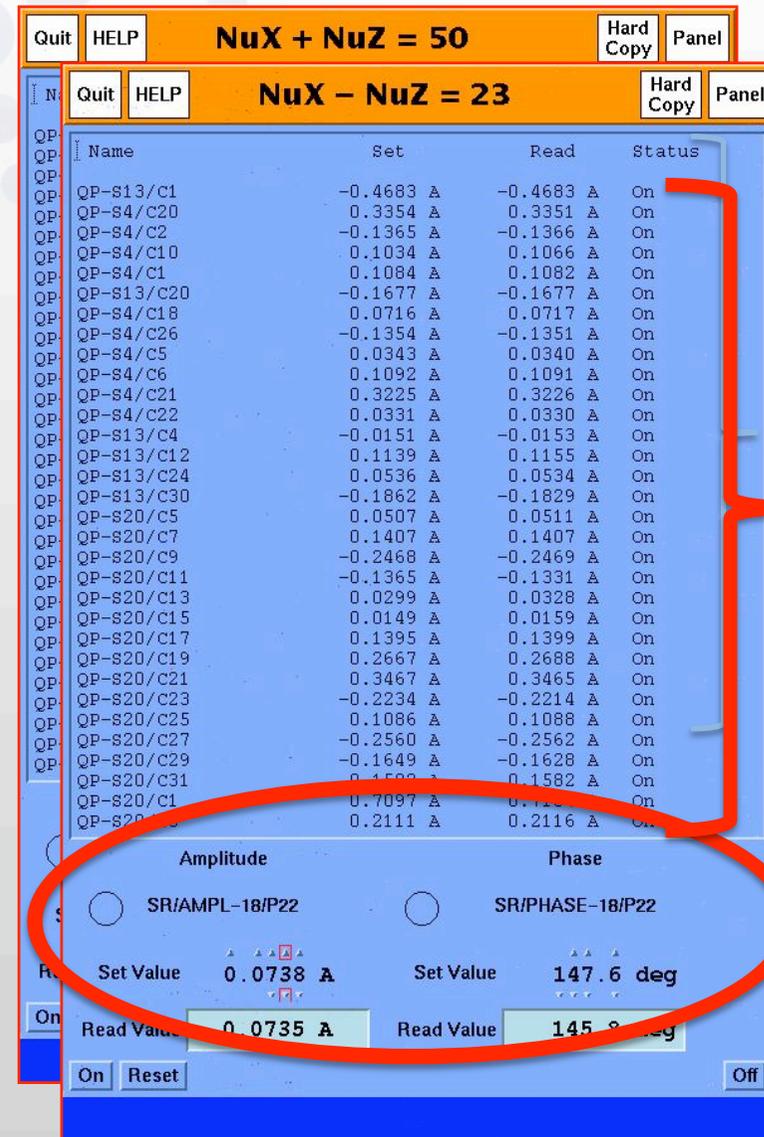
•H-V
so to

oled

•Look-up tables (corrector currents Vs ID gap aperture) were defined so to preserve the vertical emittance at any gap value.

Preserving small vertical emittance during beam delivery

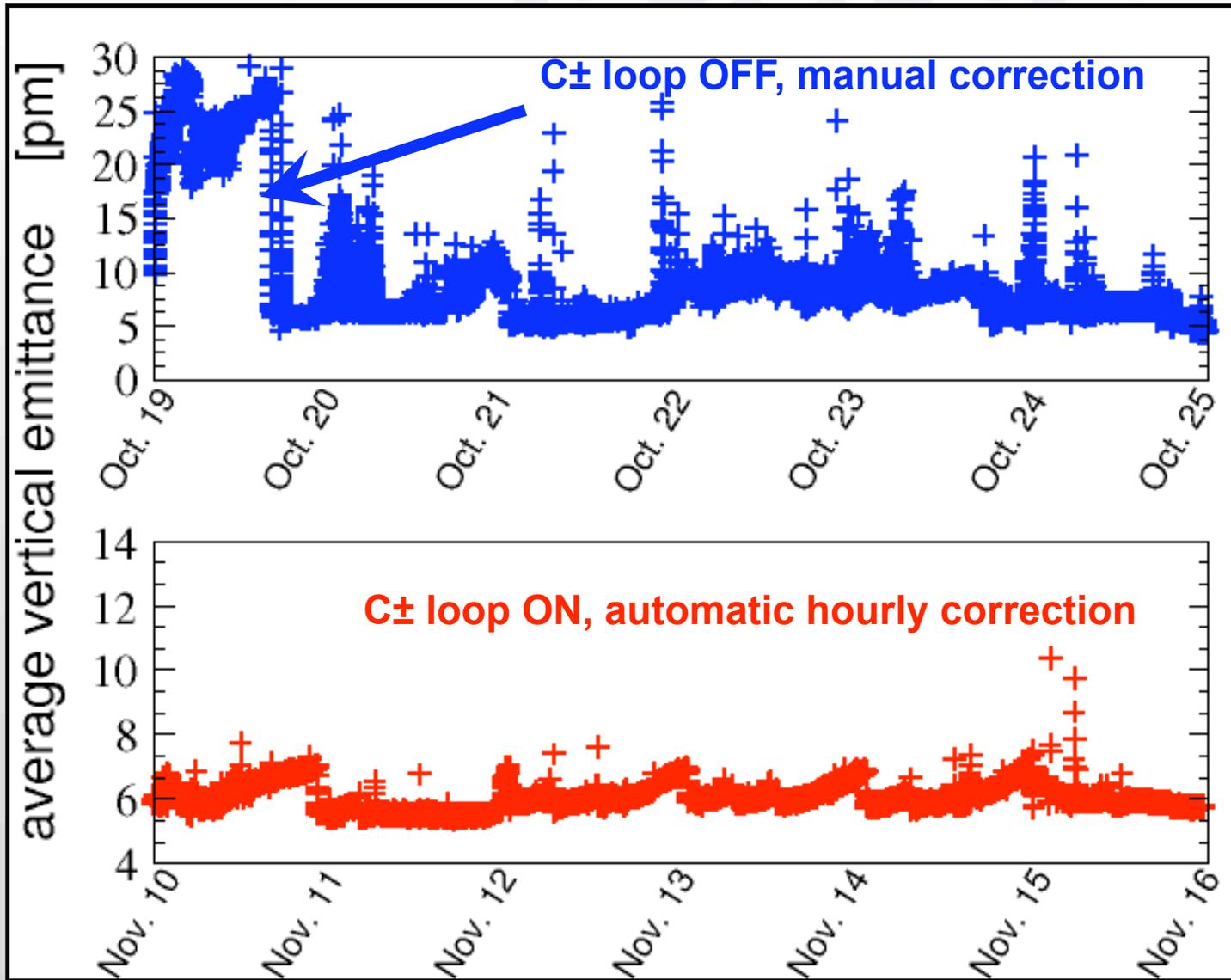
- Coupling may be represented by two complex vectors (for the sum and difference resonances respectively) $C_{\pm} = |A_{\pm}| e^{i\phi_{\pm}}$.
- At the ESRF storage ring, on top of the RDT **static** correction, C_{\pm} may be **dynamically** varied so to catch up coupling variations induced by ID gap movements.
- A new software minimizes **automatically** C_{\pm} by looking at the average vertical emittance



The screenshot shows a control interface for the NuX + NuZ = 50 system. It features a table of 32 correctors and skew quads, each with columns for Name, Set, Read, and Status. A red bracket on the right side of the table indicates that these 32 elements are correctors skew quads. Below the table, there are control fields for Amplitude and Phase, with a red oval highlighting the SR/AMPL-18/P22 and SR/PHASE-18/P22 settings. The Amplitude field shows a Set Value of 0.0738 A and a Read Value of 0.0735 A. The Phase field shows a Set Value of 147.6 deg and a Read Value of 145.8 deg.

32 correctors skew quads

Preserving small vertical emittance during beam delivery



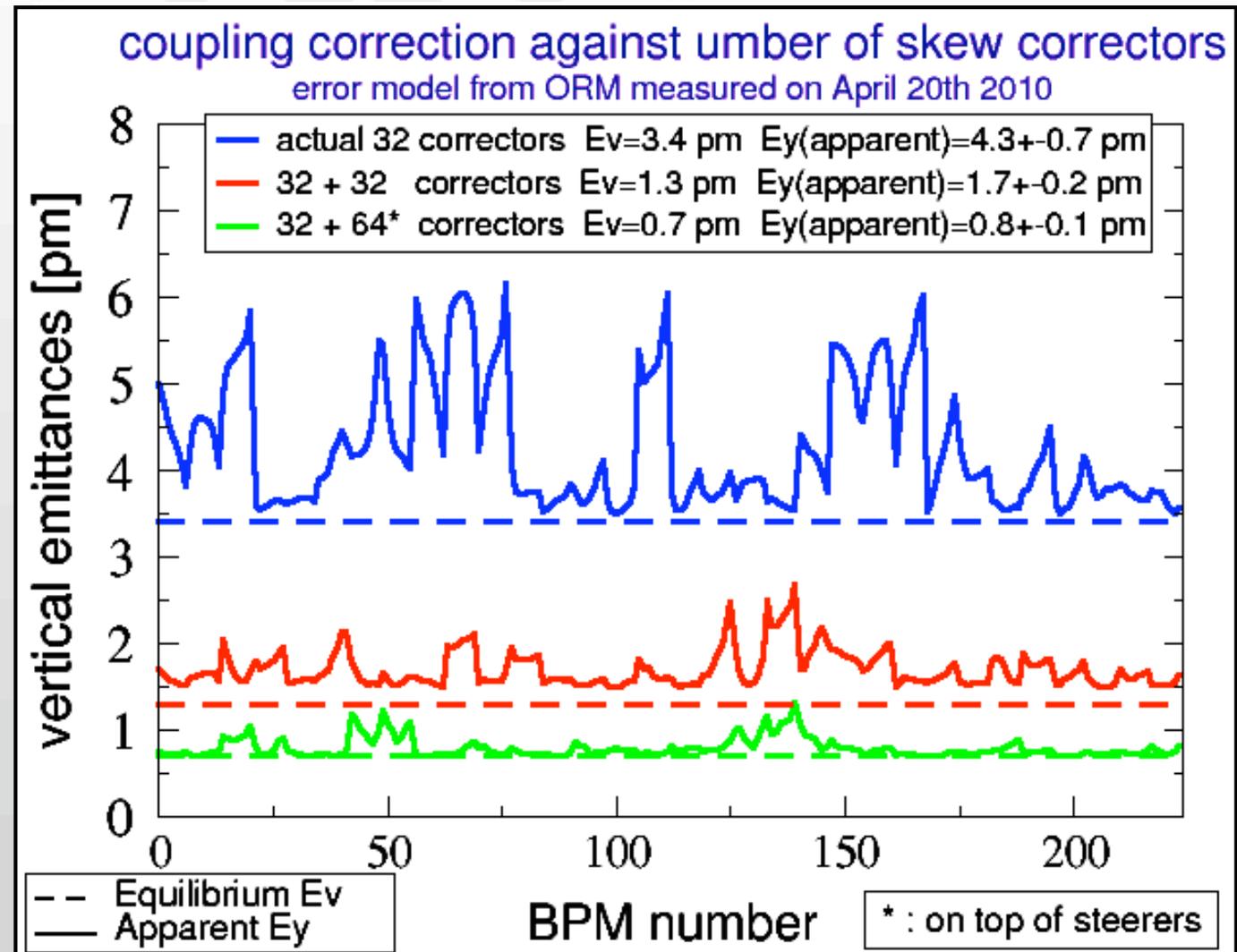
Read	Status
-0.4683 A	On
0.3351 A	On
-0.1366 A	On
0.1066 A	On
0.1082 A	On
-0.1677 A	On
0.0717 A	On
-0.1351 A	On
0.0340 A	On
0.1091 A	On
0.3226 A	On
0.0330 A	On
-0.0153 A	On
0.1155 A	On
0.0534 A	On
-0.1829 A	On
0.0511 A	On
0.1407 A	On
-0.2469 A	On
-0.1331 A	On
0.0328 A	On
0.0159 A	On
0.1399 A	On
0.2688 A	On
0.3465 A	On
-0.2214 A	On
0.1088 A	On
-0.2562 A	On
-0.1628 A	On
0.1582 A	On
0.1711 A	On
0.2116 A	On

Phase
SR/PHASE-18/P22
ue 147.6 deg
ue 145.8 deg

32 correctors skew quads

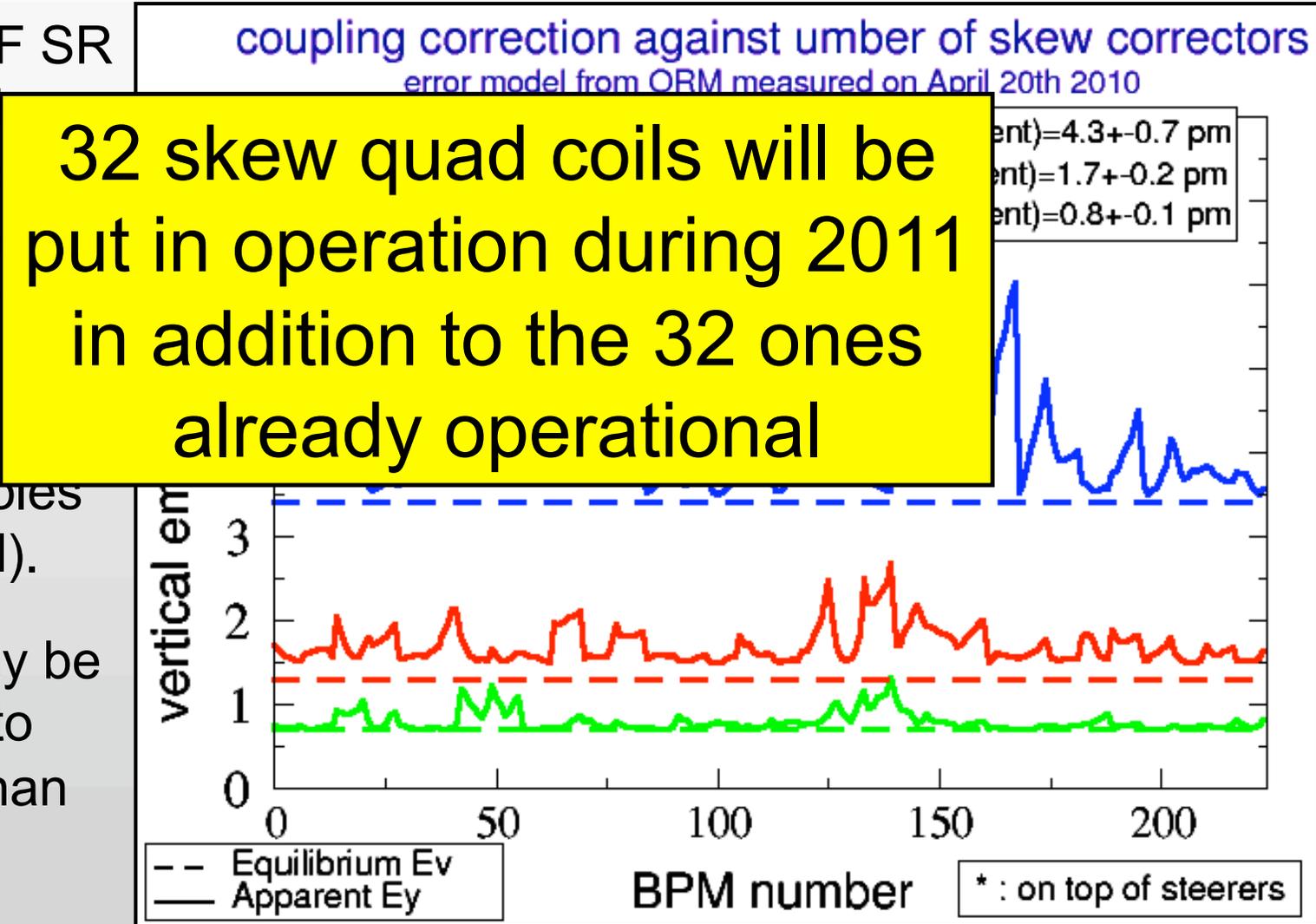
Towards ultra-small vertical emittance

- At the ESRF SR corrector coils (dipole, quad, skew quad & sextupole) are installed on the yokes of the main sextupoles (7 per 32 cell).
- 52 coils may be powered so to have more than 32 corrector skew quads.



Towards ultra-small vertical emittance

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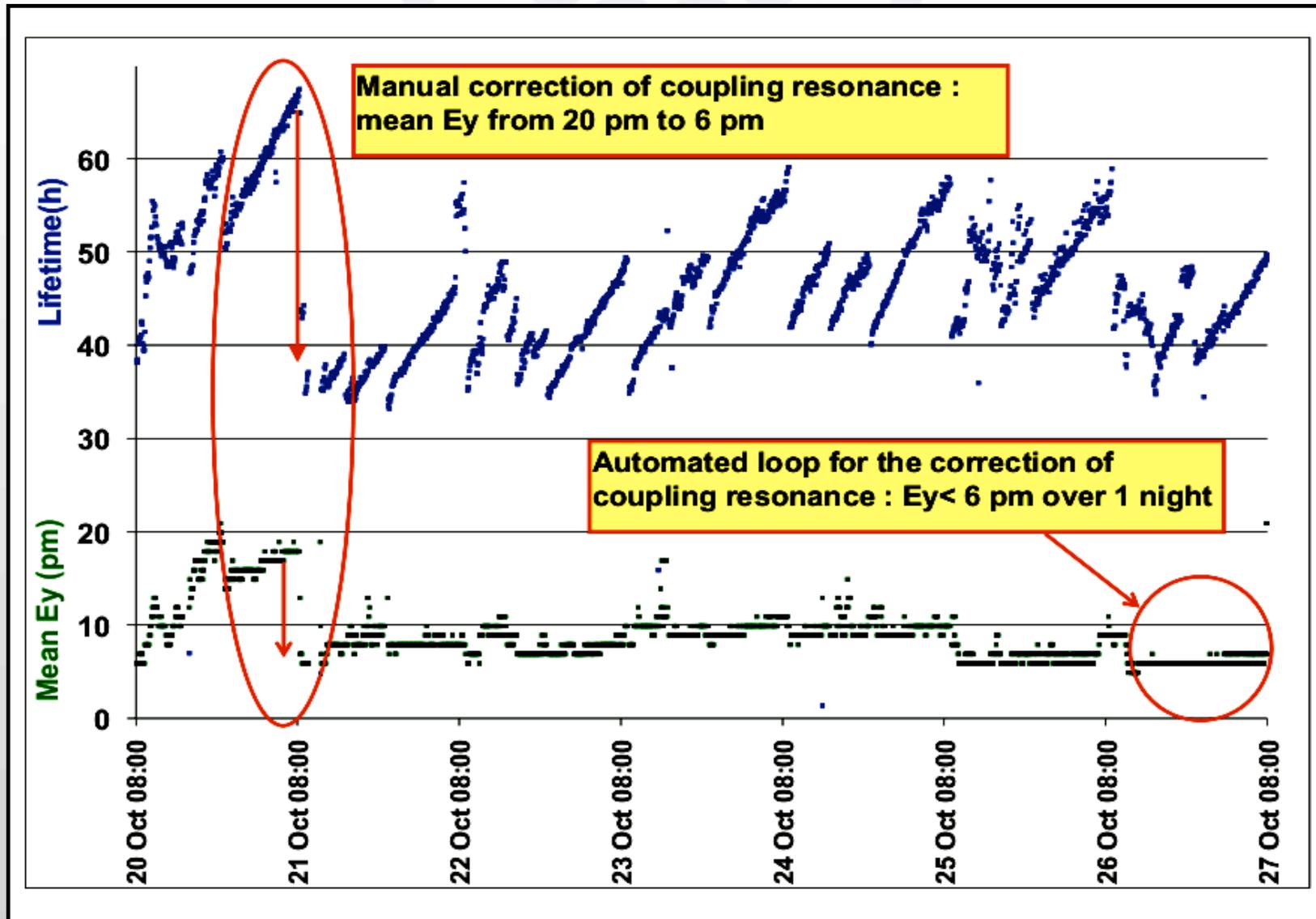


Conclusion

- The Resonance Driving Terms formalism helps clarifying the various significances of “vertical emittance” in presence of coupling and allows a straightforward, linear, correction algorithm.
- Applications to the ESRF storage ring led to vertical emittance of $\varepsilon_y = 4.4 \pm 0.7$ pm, a record low for this machine ($\varepsilon_x = 4.2$ nm $\Rightarrow \varepsilon_y/\varepsilon_x \approx 0.1\%$).
- A number of procedures to preserve small vertical emittance during beam delivery was successfully tested: stable $\varepsilon_y = 6-7$ pm (7/8 +1 filling) delivered as of November ‘10
- 32 new skew quads will be added during 2011 with the aim of delivering beam of $\varepsilon_y = 2$ pm.

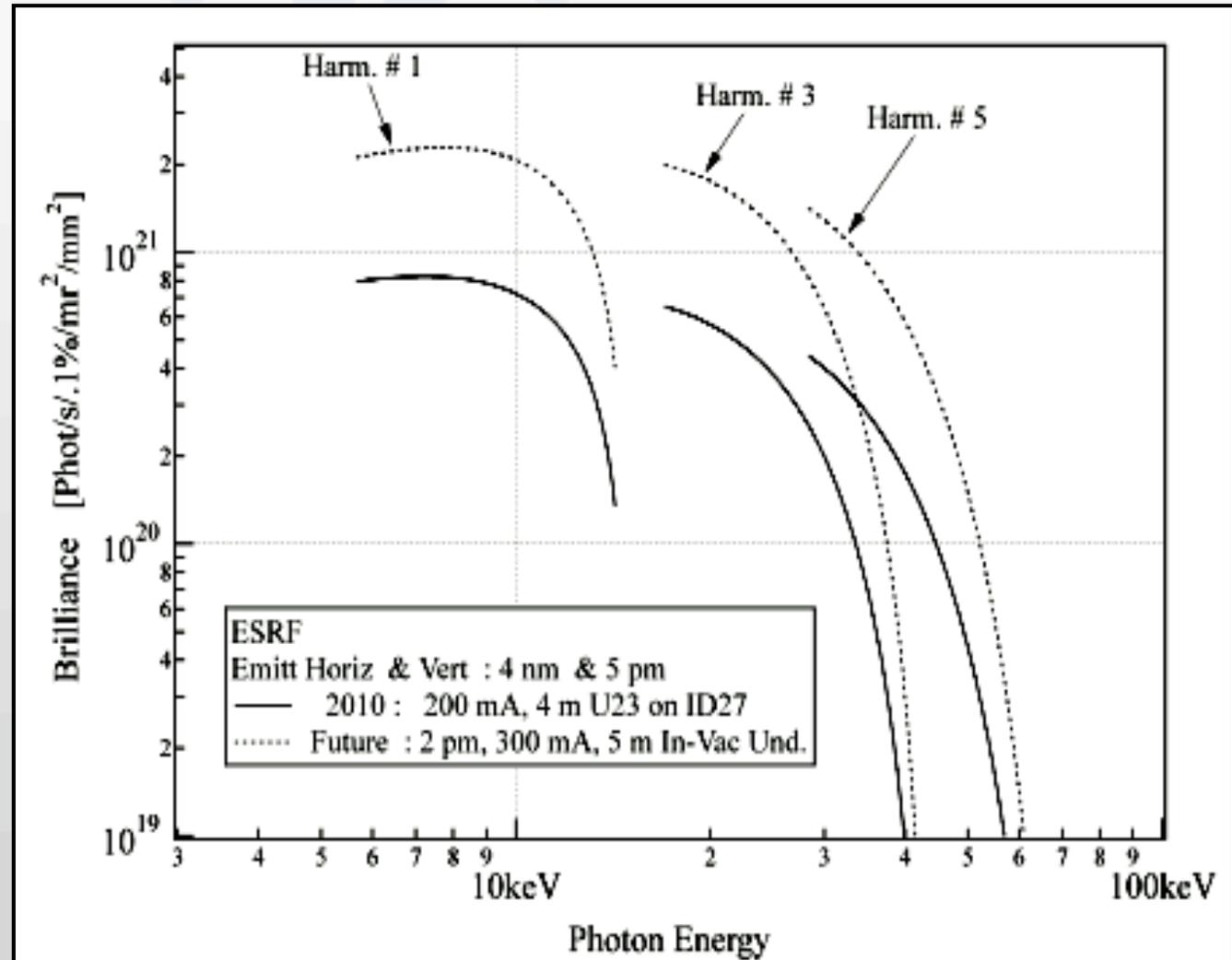
More details in a paper submitted to PRSTAB

Extra: vertical emittance Vs lifetime (7/8+1 filling)



EXTRA: Brilliance @ $\epsilon_y = 5 \text{ pm}$ @200 mA (2 pm@300mA)

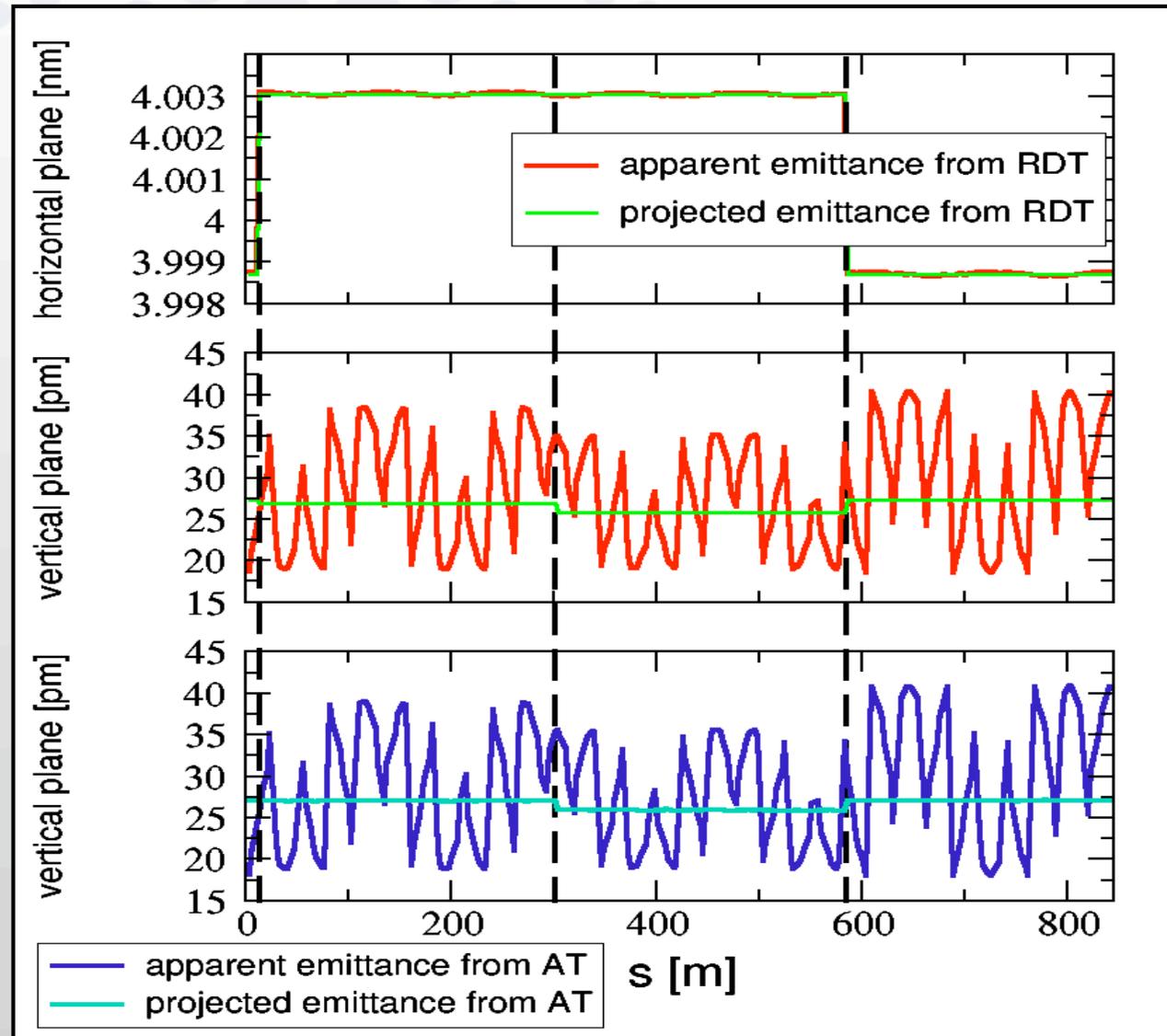
Solid curve:
Brilliance of the X-ray beam emitted from the two in-vacuum undulators installed on ID27 (High Pressure beamline). Each undulator segment has a period of 23 mm, a length of 2 m and is operated with a minimum gap of 6 mm.



EXTRA: comparing RDT formula and optics codes

- ESRF lattice with three sources of coupling only (black dashed lines): apparent and projected emittances along the ring, from RDT formulas and AT (Ohmi's formalism)

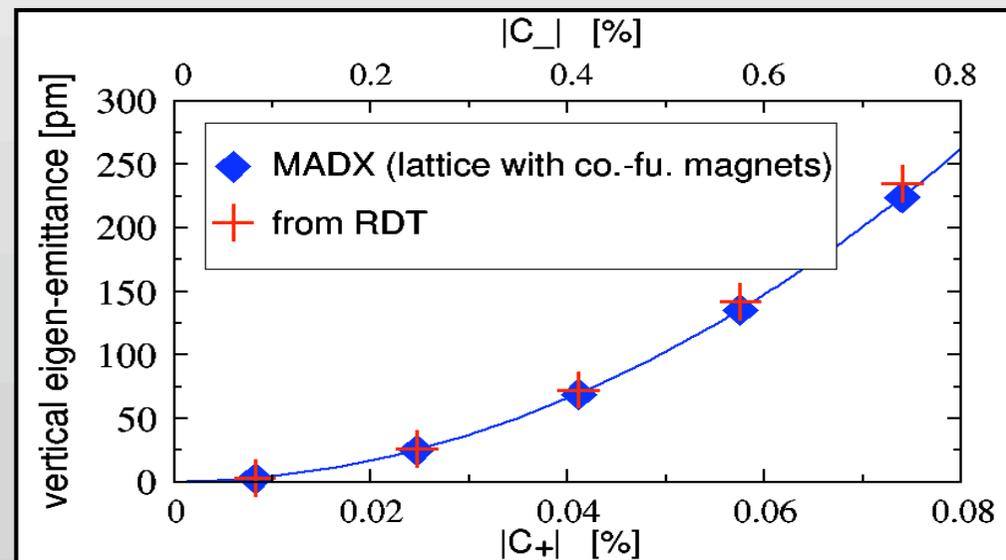
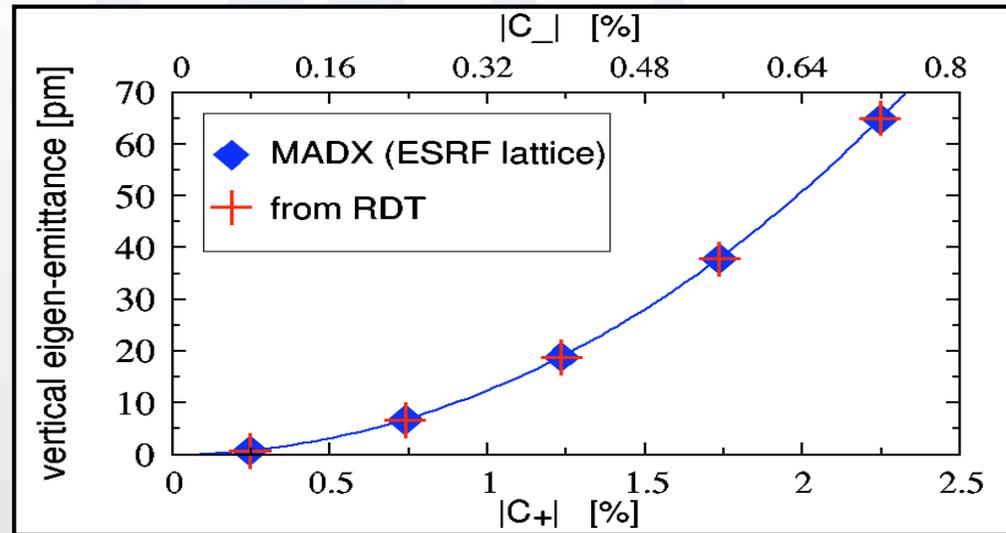
Agreement < 0.1%



EXTRA: comparing RDT formula and optics codes

- ESRF lattice (top) and lattice with quadrupole fields in bending magnets (bottom): comparing vertical eigen-emittance between RDT formula and MADX (Chao's formalism)

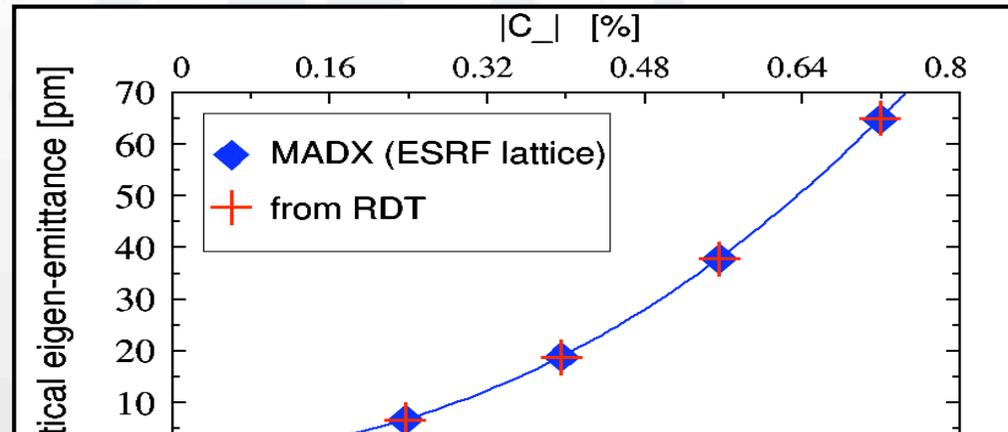
Agreement < 1% (top),
~5% (bottom)



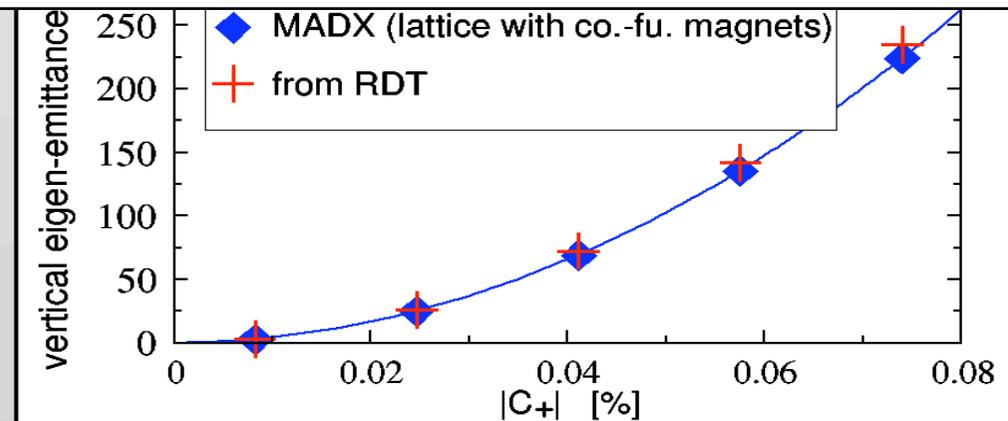
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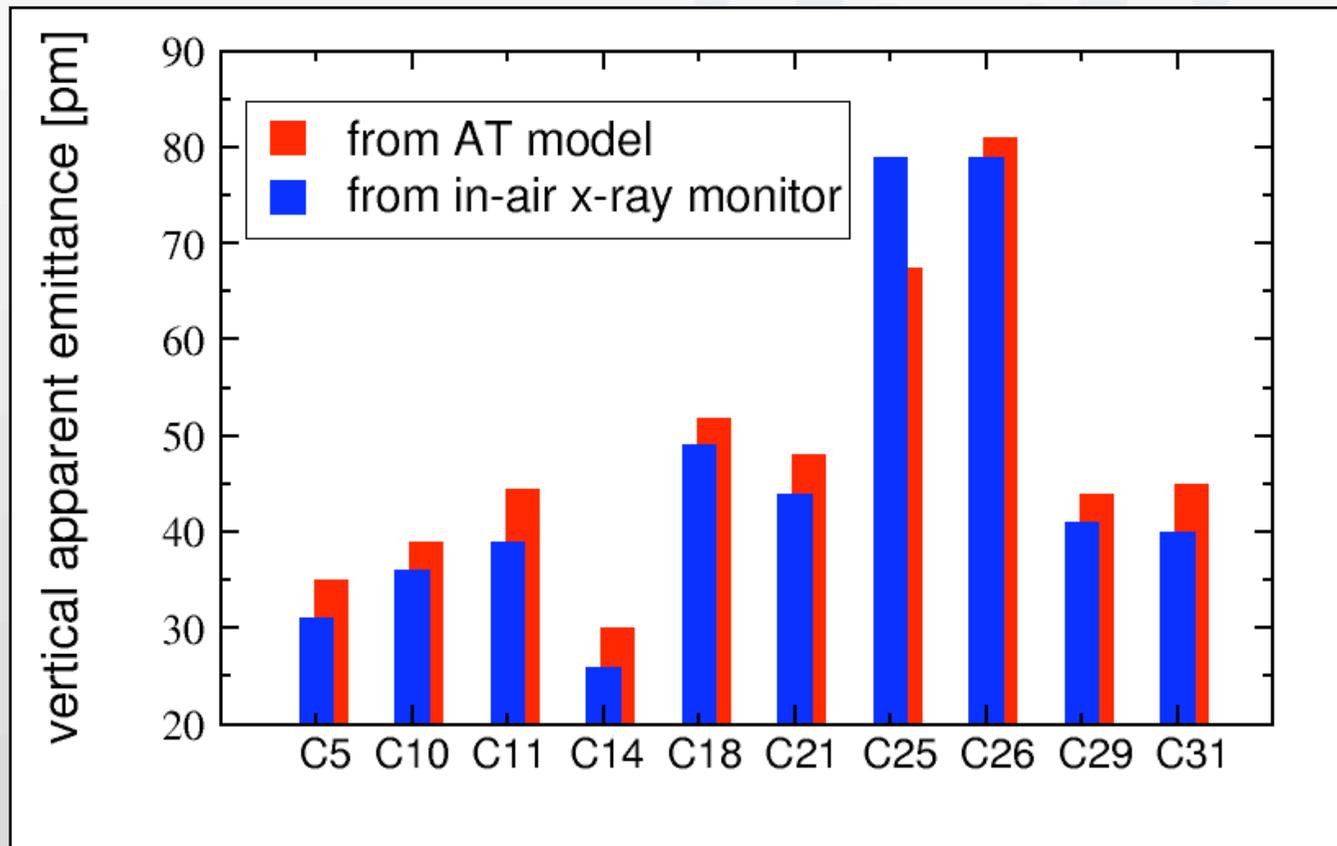
Agreement < 1% (top), ~5% (bottom)



$$\mathcal{E}_v = \frac{1}{2} \frac{\oint d(s) \{ C^2 \mathcal{H}_y^2(s) + [S_-^2 + S_+^2] \mathcal{H}_x^2(s) \} ds}{\oint \{ b_{RF}(s) - C^2 D_y(s) b_{\delta y}(s) - [S_-^2 - S_+^2] D_x(s) b_{\delta x}(s) \} ds}$$



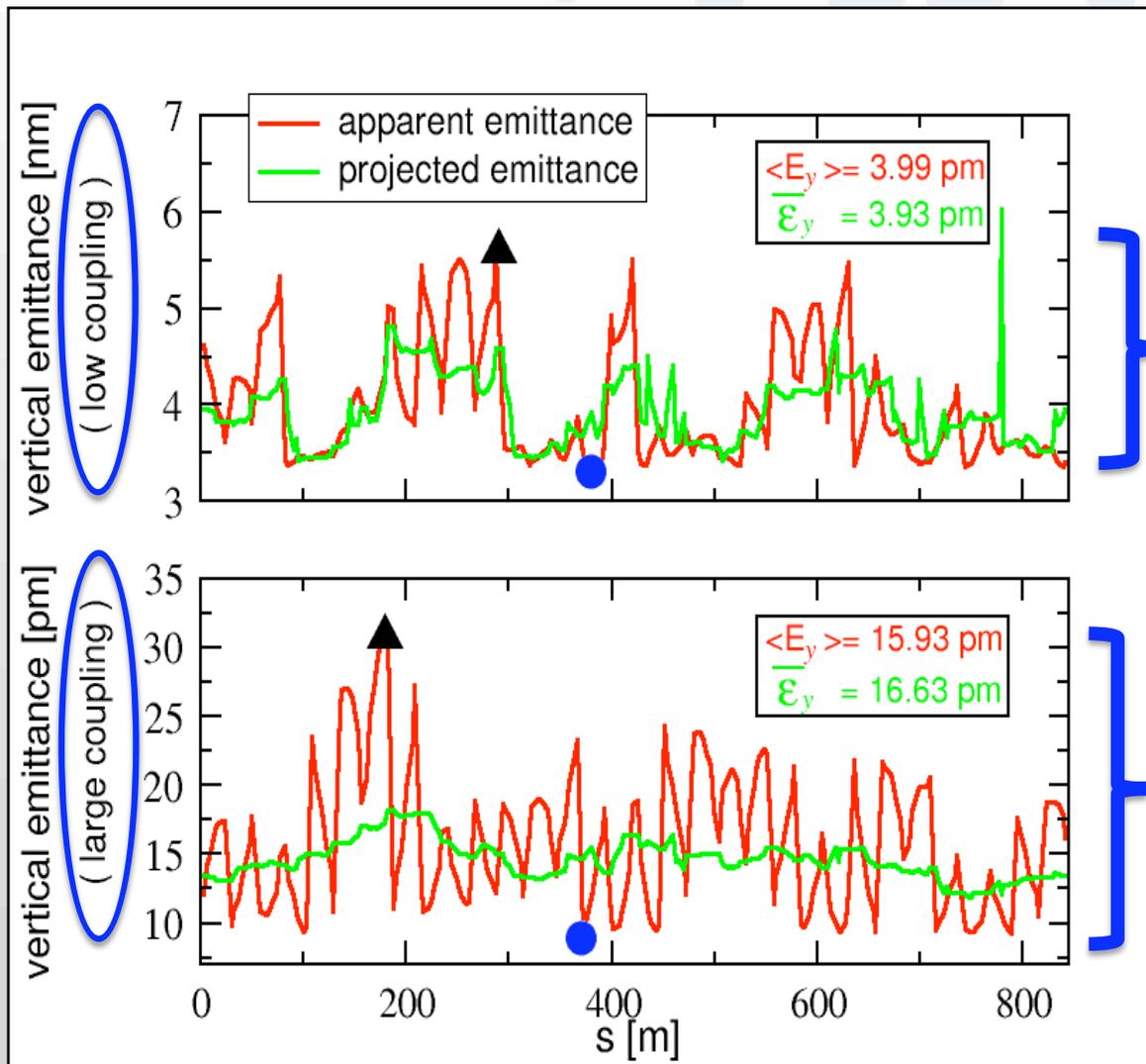
EXTRA: Vertical apparent emittance “spread”



Measurement of Jan. 16th 2010, before correction

- The larger the coupling, the larger is the “*spread*” among the **measured apparent** vertical emittance along the ring (don’t blame “bad” emittance monitors for larger-than-expected measured values)

EXTRA: Vertical apparent emittance “spread”

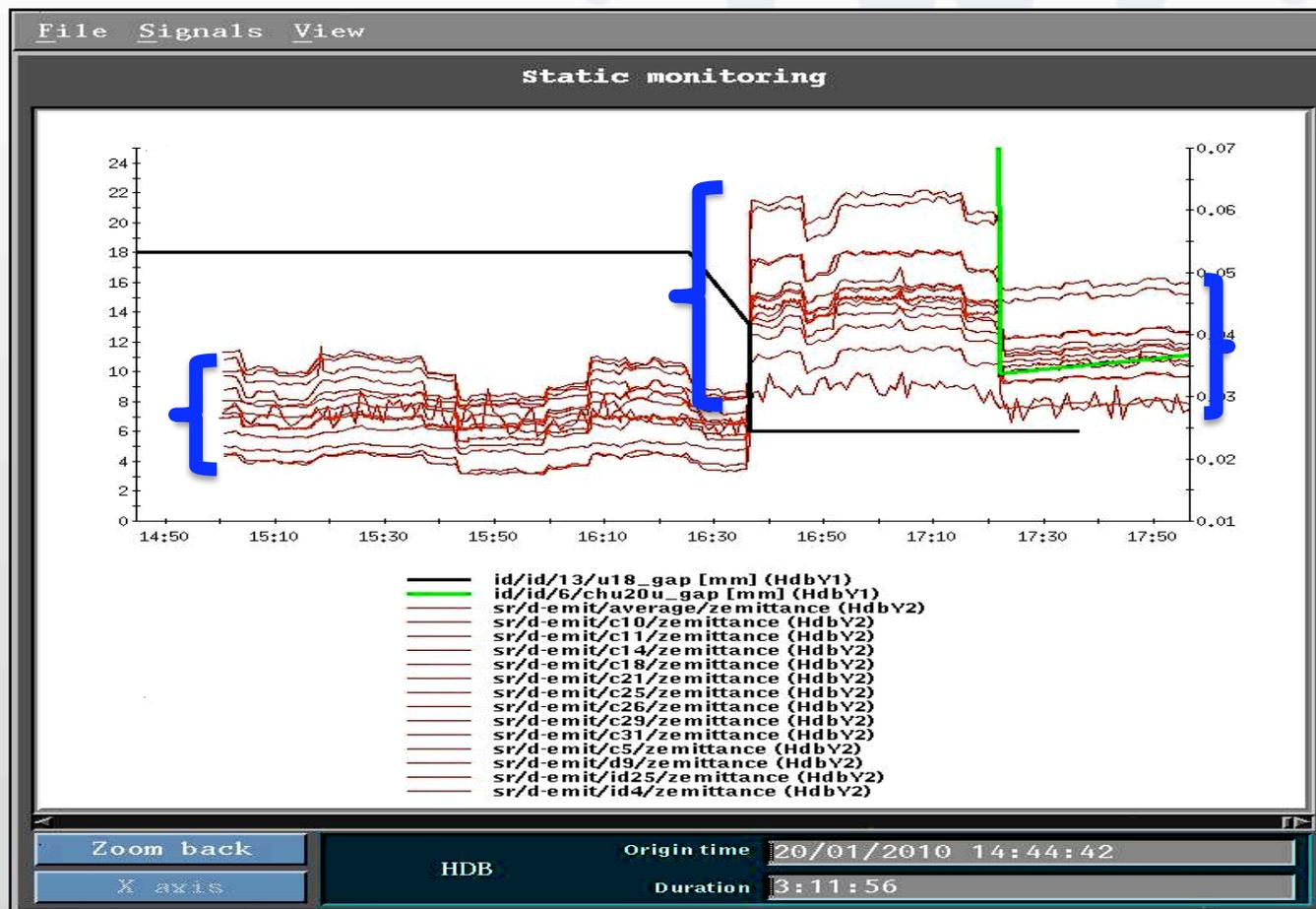


$\delta \varepsilon_y = 2.5 \text{ pm}$

$\delta \varepsilon_y = 20 \text{ pm}$

- The larger the coupling, the larger is the “*spread*” among the **measured apparent** vertical emittance along the ring (don’t blame “bad” emittance monitors for larger-than-expected measured values)

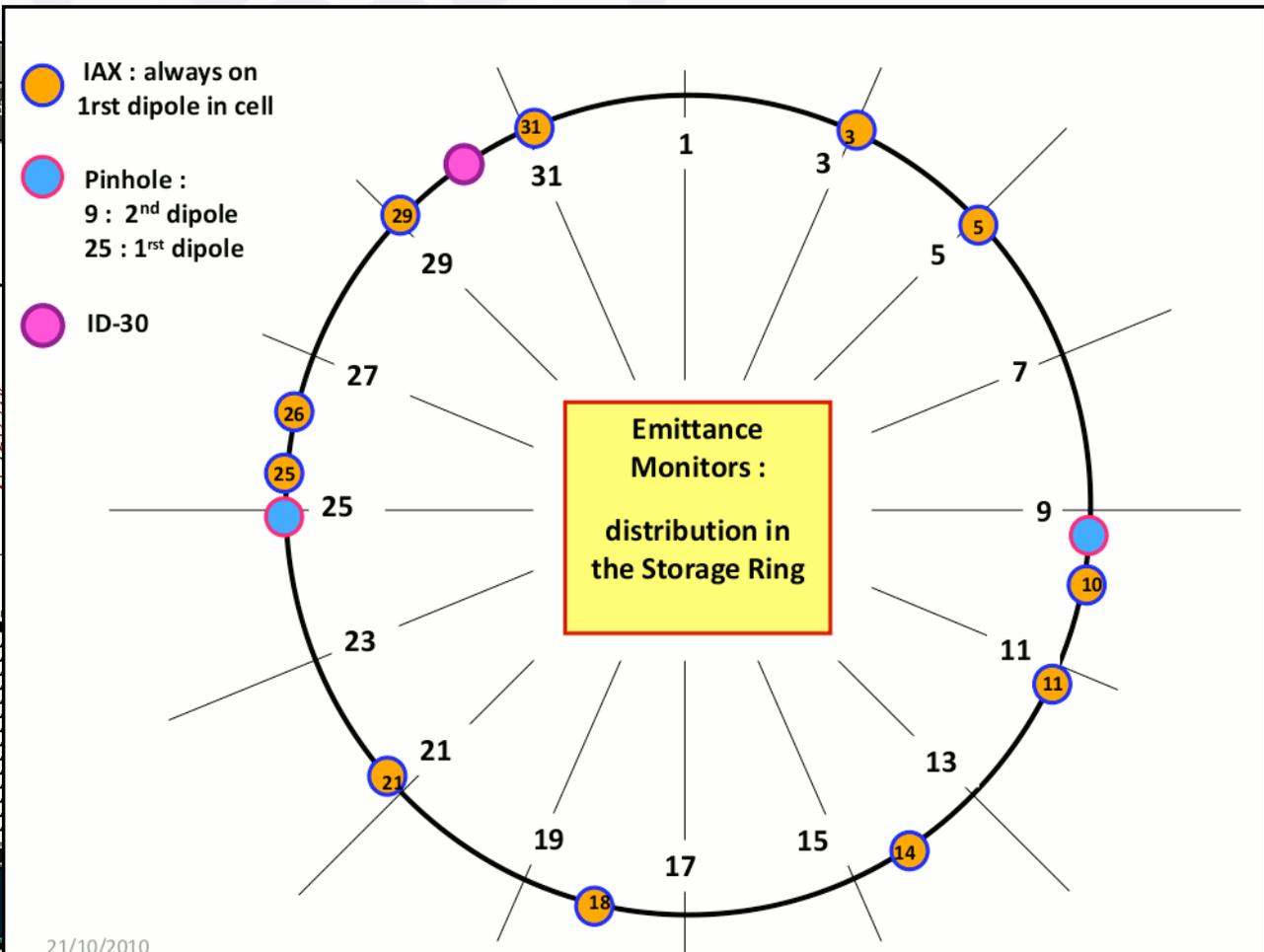
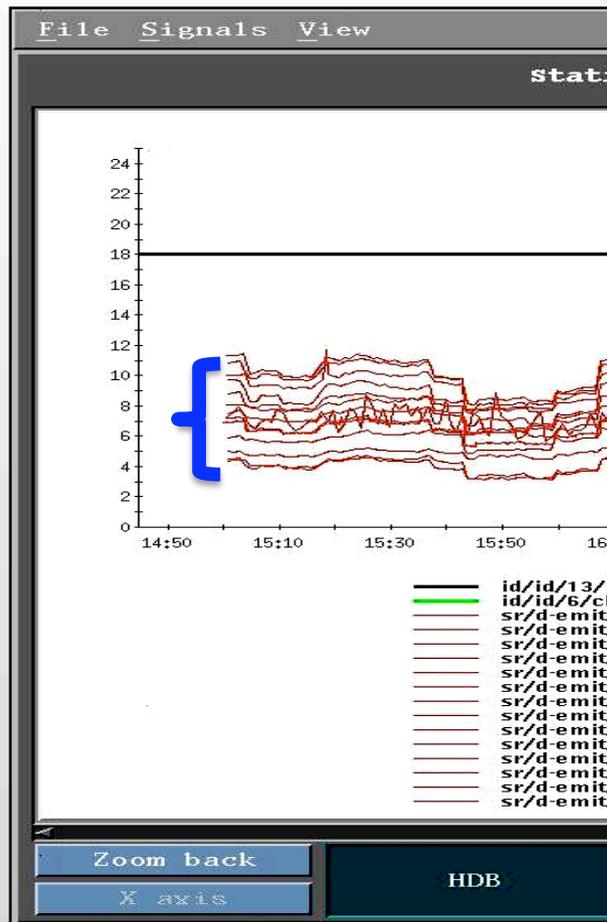
EXTRA: Vertical apparent emittance “spread”



Apparent emittance measured at 14 monitors on Jan. 20th 2010, during beam delivery and movements of two ID gaps movements

- The larger the coupling, the larger is the “*spread*” among the **measured apparent** vertical emittance along the ring (don’t blame “bad” emittance monitors for larger-than-expected measured values)

EXTRA: vertical emittance monitors in the ESRF SR



Apparent emittance measured at 14 monitors on Jan. 20th 2010, during beam delivery and two ID gaps movements

expected measured values)

EXTRA: ESRF SR correctors matrix

- At the ESRF SR corrector coils (dipole, quad, skew quad & sextupole) are installed on the yokes of the main sextupoles (7 per 32 cell).
- 52 coils may be powered so to have more than 32 corrector skew quads.

cell \ sext.	S04	S06	S13	S19	S20	S22	S24
4	norm. quad	steerer	skew quad	steerer	norm. quad	steerer	
5	skew quad	steerer		steerer	skew quad	steerer	norm. quad
6	skew quad	steerer	norm. quad	steerer		steerer	
7		steerer		steerer	skew quad	steerer	norm. quad
8		steerer	norm. quad	steerer	norm. sext	steerer	norm. sext
9		steerer	norm. quad	steerer	skew quad	steerer	norm. sext
10	skew quad	steerer		steerer		steerer	norm. quad
11		steerer	norm. quad	steerer	skew quad	steerer	
12		steerer	skew quad	steerer	norm. quad	steerer	
13		steerer	norm. quad	steerer	skew quad	steerer	
14	norm. quad	steerer		steerer		steerer	norm. quad
15		steerer	norm. quad	steerer	skew quad	steerer	
16		steerer		steerer	norm. sext	steerer	norm. sext
17		steerer	norm. quad	steerer	skew quad	steerer	
18	skew quad	steerer		steerer		steerer	norm. quad
19		steerer	norm. quad	steerer	skew quad	steerer	
20	skew quad	steerer	skew quad	steerer	norm. quad	steerer	
21	skew quad	steerer	norm. quad	steerer	skew quad	steerer	
22	skew quad	steerer	norm. quad	steerer		steerer	
23	norm. quad	steerer		steerer	skew quad	steerer	norm. sext
24		steerer	skew quad	steerer	norm. sext	steerer	norm. sext
25		steerer	norm. quad	steerer	skew quad	steerer	
26	skew quad	steerer		steerer		steerer	norm. quad
27		steerer	norm. quad	steerer	skew quad	steerer	
28	norm. quad	steerer		steerer		steerer	norm. quad
29		steerer	norm. quad	steerer	skew quad	steerer	norm. sext
30		steerer	skew quad	steerer	norm. quad	steerer	
31		steerer	norm. quad	steerer	skew quad	steerer	
32		steerer	norm. quad	steerer	norm. sext	steerer	norm. sext
1	skew quad	steerer	skew quad	steerer	skew quad	steerer	
2	skew quad	steerer		steerer		steerer	norm. quad
3		steerer	norm. quad	steerer	skew quad	steerer	norm. sext

96 steerers

32 norm. quads

32 skew quads

12 norm. sext.s

Free channels:
52