Partial coherence in undulator beamlines at ultra-low emittance storage rings

Manuel Sanchez del Rio

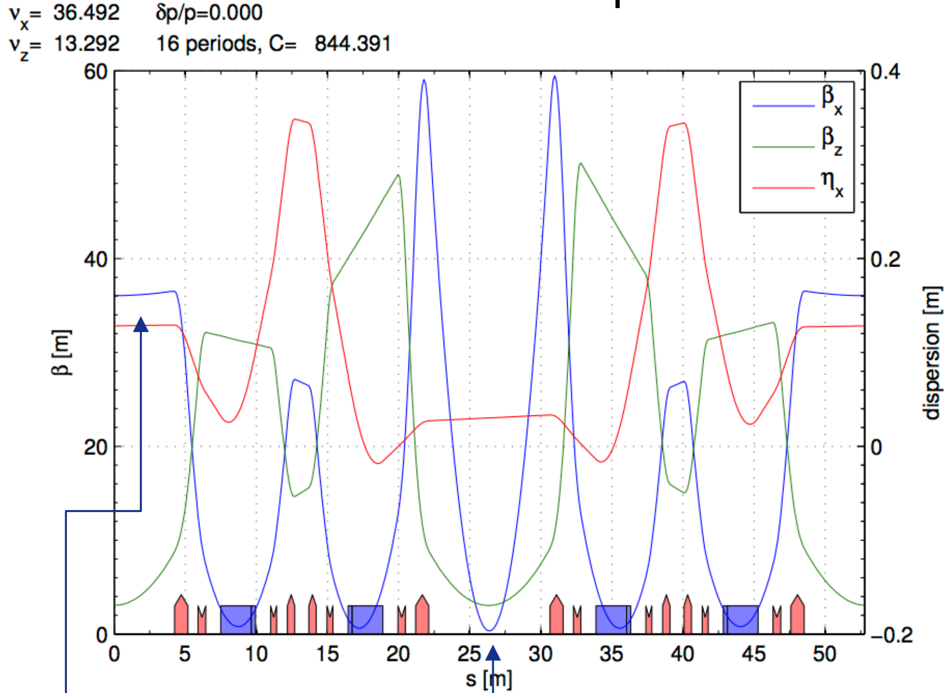
OUTLOOK

MOTIVATION:

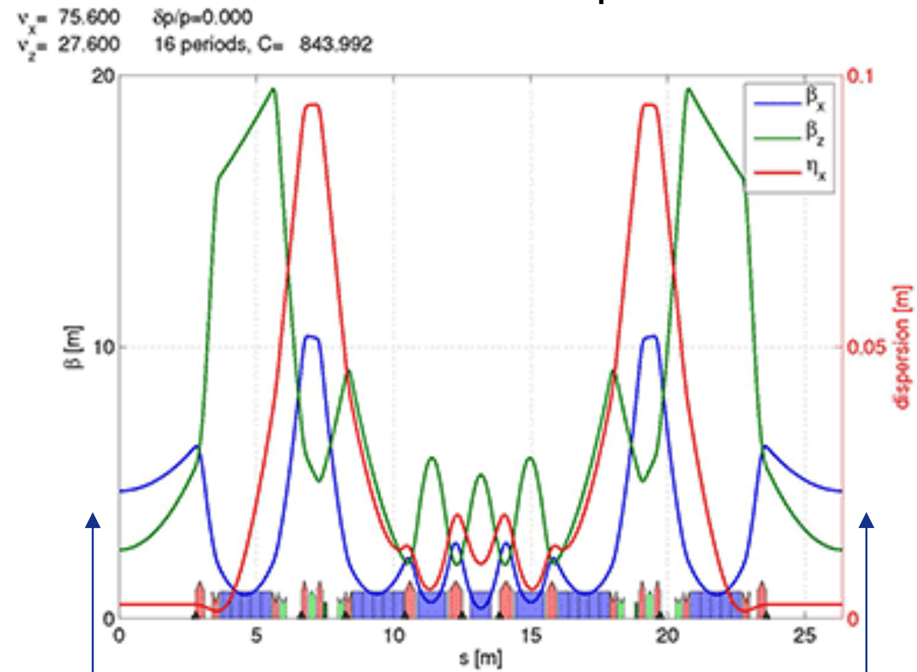
Fully characterize and calculate coherence properties of EBS beamlines.

- Introduction (with a bit of theory)
- Methodology: Coherent mode decomposition.
- Results for
  - ESRF: comparison Hβ-Lβ vs new EBS
  - ID16A

Horizontal emittance = 4000 pm



Horizontal emittance = 137 pm



High Beta:
β_x = 37.59 m
β_y = 2.95 m
σ_x = 415.0 μm
σ_y = 3.43 μm
$\sigma_{x'}$ = 10.3 μrad
$\sigma_{z'}$ =1.16 μrad

Low Beta:
β_x = 0.35 m
β_y = 3.0 m
σ_x = 50.4 μm
σ_y = 3.44 μm
$\sigma_{x'}$ = 107.2 μrad
$\sigma_{z'}$ =1.16 μrad

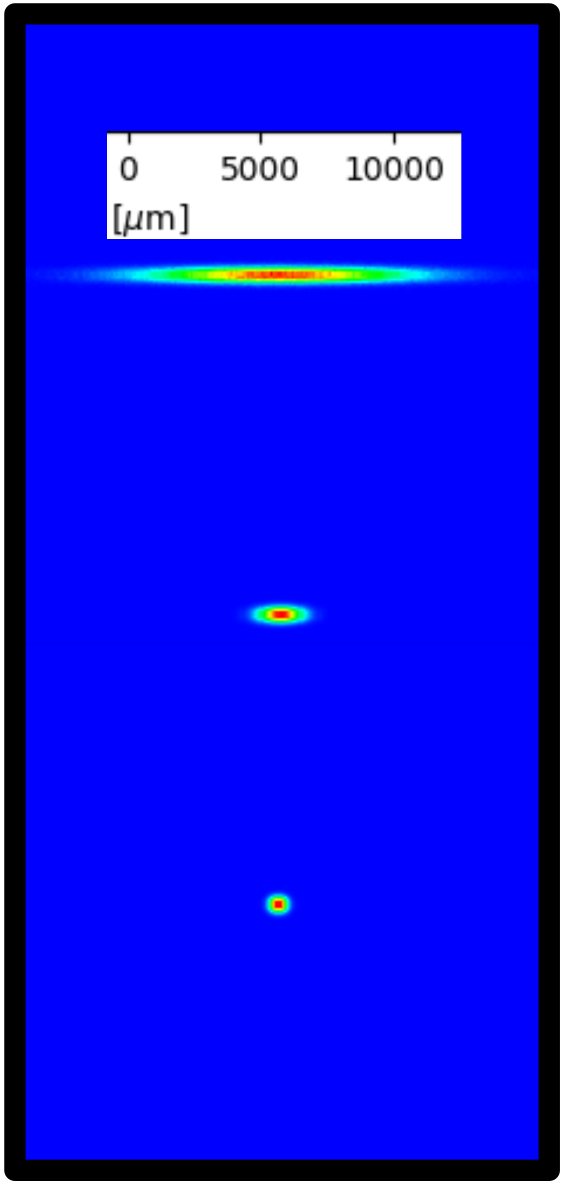
EBS (S28D):
β_x = 6.90 m
β_y = 2.64 m
σ_x = 30.2 μm
σ_y = 3.64 μm
$\sigma_{x'}$ = 4.37 μrad
$\sigma_{z'}$ =1.37 μrad

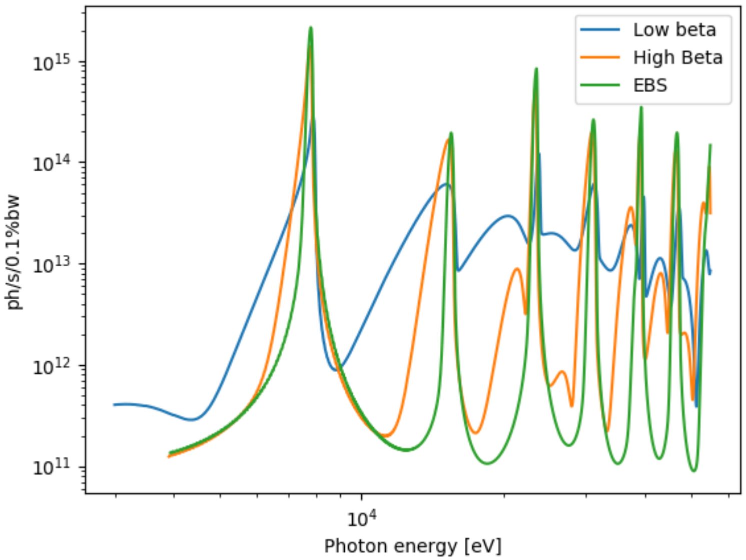
EBS – ESRF U18 2m @ 8 keV L=2m observed at 30m

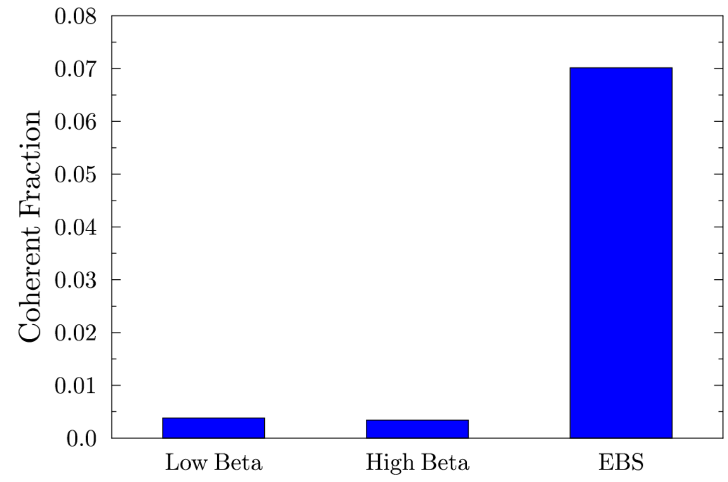
Low Beta U18

High Beta U18

EBS U18







FROM THEORY TO SOFTWARE

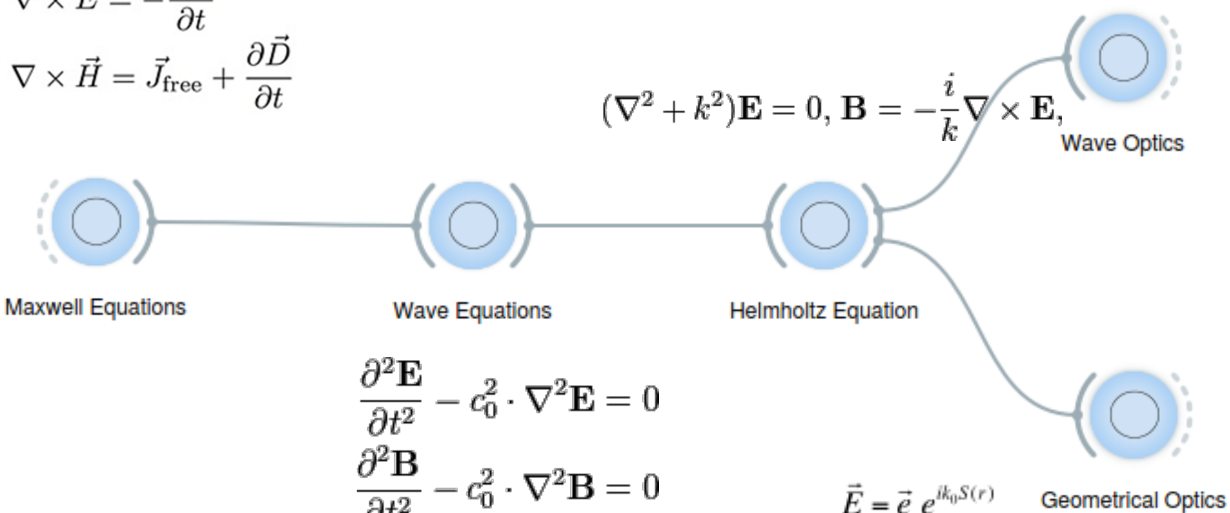
$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

$$(\nabla^2 + k^2)\mathbf{E} = 0,\ \mathbf{B} = -\frac{i}{k}\nabla \times \mathbf{E},$$



$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

$$\vec{E} = \vec{e}\ e^{ik_0 S(r)}$$

$$\vec{H} = \vec{h}\ e^{ik_0 S(r)}$$

$$(\nabla S)^2 = n^2$$

$$\nabla S = n\ \vec{s}$$

$$\frac{d}{ds}\left(n \frac{d\vec{r}}{ds}\right) = \nabla n$$

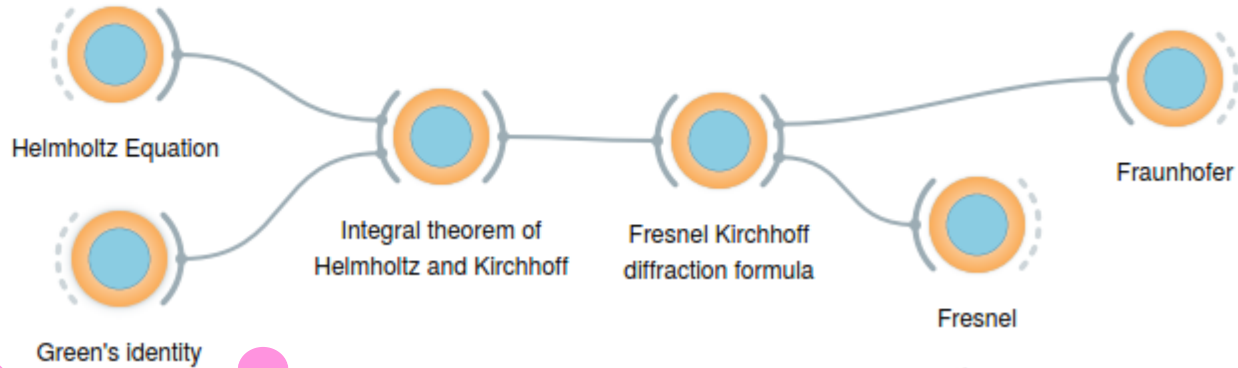
$$\nabla n = 0 \Rightarrow \frac{d\vec{r}}{ds} = 0 \Rightarrow \vec{r} = s\vec{a} + \vec{b}$$

- Wofry Wavefront Propagation
- Wofry Beamline Elements
- Wofry Tools
- SRW Light Sources
- SRW Optical Elements
- SRW Tools
- SRW Wofry
- WISE
- WISE Tools
- WISE Wofry
- COMSYL

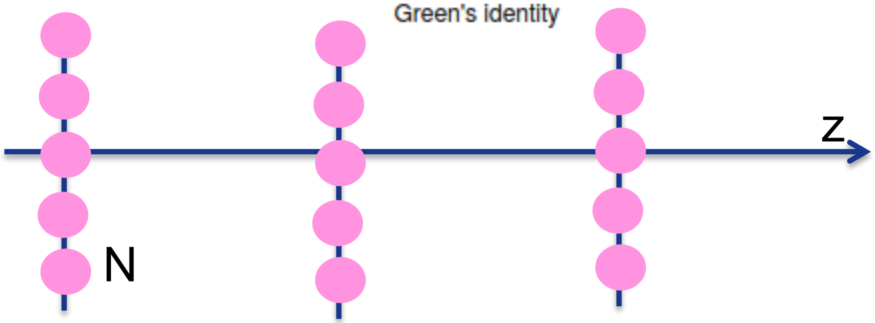
OrAnge SYnchrotron Suite
OASYS
Powered by orange

- Shadow Sources
- Shadow Optical Elements
- Shadow Compound Optical ...
- Shadow Special Elements
- Shadow PostProcessor
- Shadow PreProcessor
- Shadow Experiments
- Shadow Loop Management
- Shadow Utility

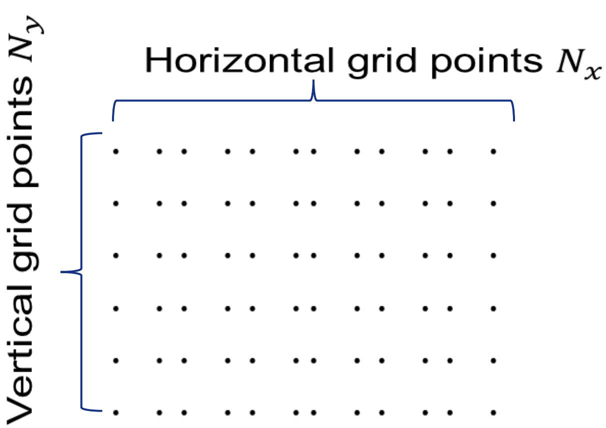
Todo
Input plane
aliasing



$$E(\tilde{\mathbf{r}}, z, \omega) = \frac{e^{ikz}}{i\lambda z}\int_A d\mathbf{r}' A(\mathbf{r}') e^{\frac{ik}{2z}(\tilde{\mathbf{r}}-\mathbf{r}')^2}.$$



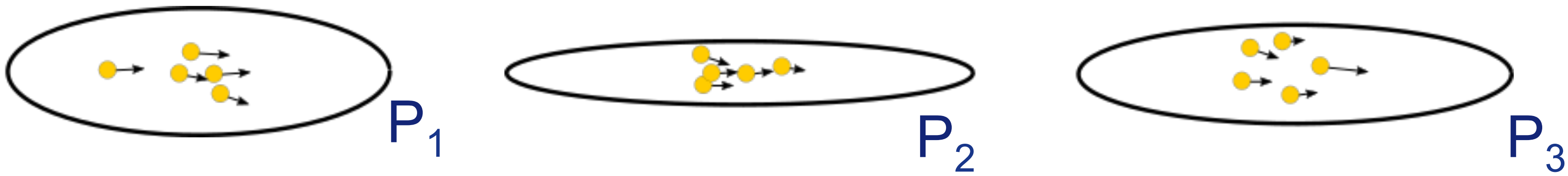
$$E'(\boldsymbol{r}) = \int h(\boldsymbol{r},\boldsymbol{r}',\omega)E(\boldsymbol{r}',\omega)d\boldsymbol{r}'$$



Estimation of operations:
- $(N_x \times N_y)^2$ operations: $N_x N_y$ integrals
- $N_x \times N_y$ with FTT (Fourier Optics)

for $N \sim 10^3$
- Both integral and Fourier methods OK in 1D
- Only Fourier methods in 2D

Statistically distributed bunches

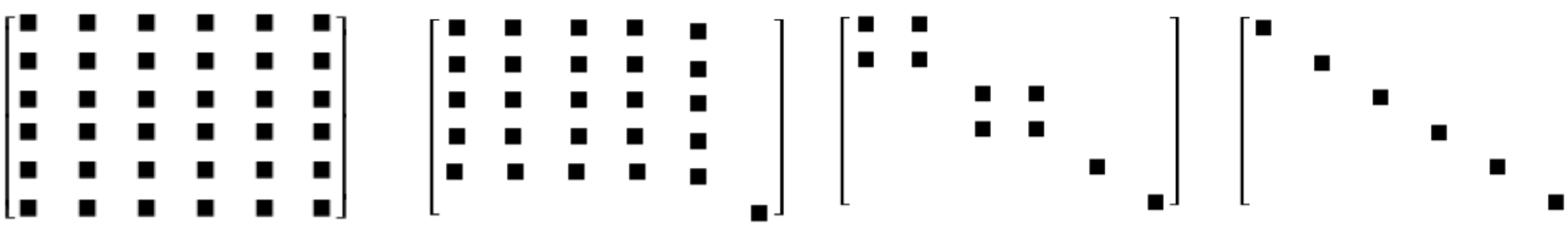


Order of 10^9 electrons per bunch

$$f(\mathbf{r},\boldsymbol{\theta},\gamma,z) = C \cdot \exp\left(-\mathbf{u}^{\mathrm{T}}\Sigma^{-1}\boldsymbol{u}\right)$$

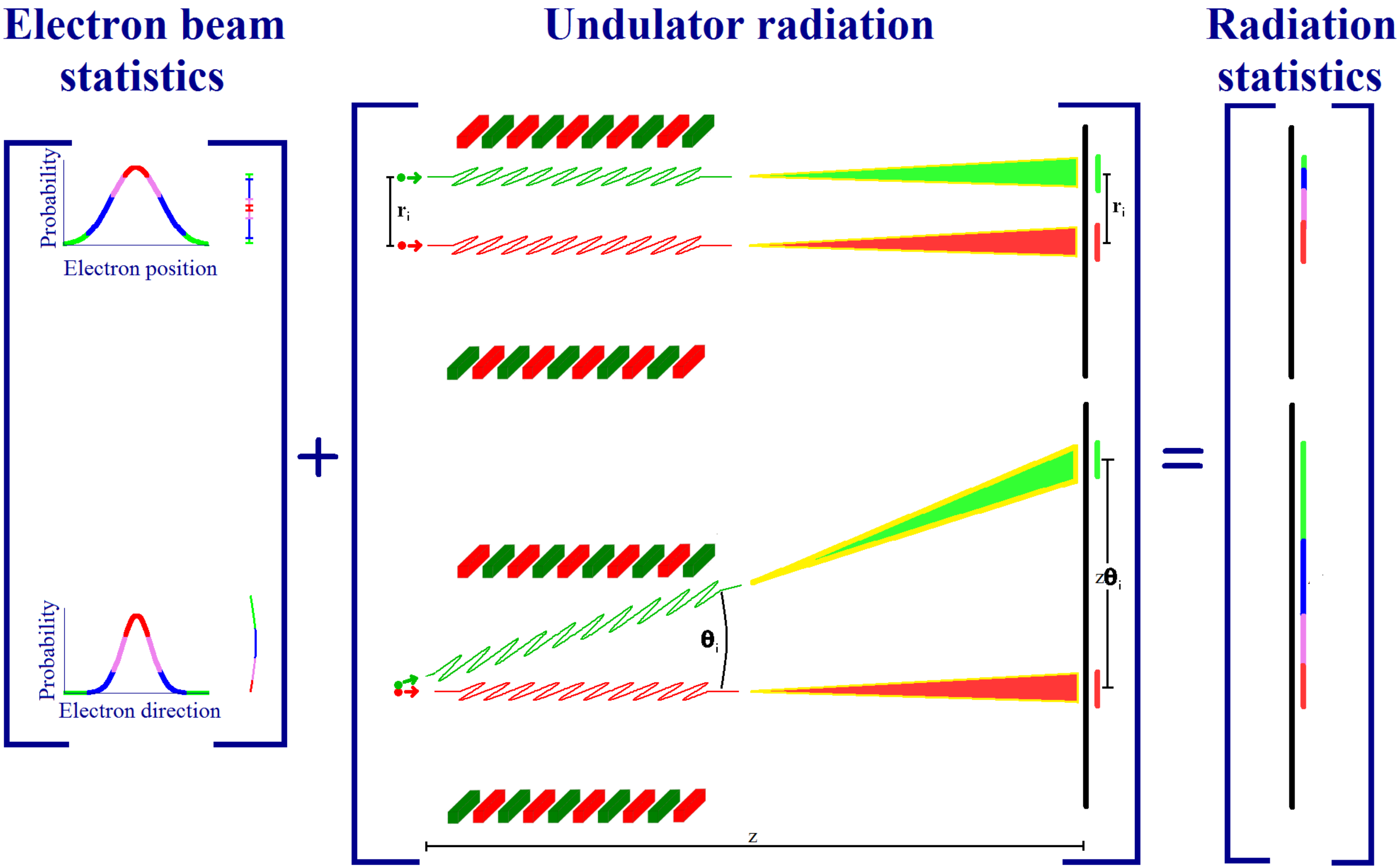
Phase space vector $\boldsymbol{u} = (x, \theta_x, y, \theta_y, \gamma, z)$

6x6 covariance matrix Σ:



General | No z-coupling | Finite alpha | Symmetry point

beam statistics → statistics of the emission



Kim, K.-J. Proc. SPIE 0582 (1986)

Mutual coherence function $\Gamma(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2)\rangle_e$

$$\left(\Delta_{\boldsymbol{r}_1} - \frac{1}{c^2}\frac{\partial^2}{\partial t_1^2}\right)\left(\Delta_{\boldsymbol{r}_2} - \frac{1}{c^2}\frac{\partial^2}{\partial t_2^2}\right)\Gamma(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = (4\pi)^2 \Gamma_Q(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2)$$

$$\Gamma_Q(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \langle Q^*(\boldsymbol{r}_1, t_1) Q(\boldsymbol{r}_2, t_2)\rangle_e$$

$$Q(\boldsymbol{r}, t) = -\left(\frac{1}{\epsilon_0}\nabla\rho + \mu_0 \frac{\partial \boldsymbol{J}}{\partial t}\right)$$

Wide-sense stationary:

$$\langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2)\rangle_e = \langle E^*(\boldsymbol{r}_1, 0) E(\boldsymbol{r}_2, t_2 - t_1\ )\rangle_e$$

Storage ring emission is *wide-sense stationary* if

- the bunch length is long enough
- the radiation frequency is large enough
- the monochromator resolution is not too high

Geloni, G., et al. Nucl. Inst. and Meth. in Physics 588 463-493 (2008)

Frequency representation:

$$\langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2) \rangle_e \xleftrightarrow{\text{FT}} \langle E^*(\boldsymbol{r}_1, \omega_1) E(\boldsymbol{r}_2, \omega_2) \rangle_e$$

In consequence:

$$\langle E^*(\boldsymbol{r}_1, \omega_1) E(\boldsymbol{r}_2, \omega_2) \rangle_e \Rightarrow \underbrace{\langle E^*(\boldsymbol{r}_1, \omega) E(\boldsymbol{r}_2, \omega) \rangle_e}_{\text{much simpler}}$$

- **Cross spectral density** (CSD) **[everything]**

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \langle E_1^*(\boldsymbol{r}_1, \omega) E_2(\boldsymbol{r}_2, \omega) \rangle_e$$

- **Spectral density** (kind of “intensity” / “energy”)

$$S(\boldsymbol{r}, \omega) = W(\boldsymbol{r}, \boldsymbol{r}, \omega)$$

- **Spectral degree of coherence**

$$\mu(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \frac{W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)}{\sqrt{S(\boldsymbol{r}_1, \omega) S(\boldsymbol{r}_2, \omega)}}$$

(incoherent) $0 \le |\mu(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)| \le 1$ (comp coherent)

$W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$ **four-dimensional** for fixed frequency at a distance z.

Propagation:

$$W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \int W(\boldsymbol{r}_1', \boldsymbol{r}_2', \omega) h^*(\boldsymbol{r}_1, \boldsymbol{r}_1', \omega) h(\boldsymbol{r}_2, \boldsymbol{r}_2', \omega) d\boldsymbol{r}_1' \, d\boldsymbol{r}_2'$$

$N_x, N_y \in [100{,}1000]$.

Memory size ~ $N_x^2 N_y^2$

$100^4 = 10^8$ **to** $1000^4 = 10^{12}$

complex numbers (16 bytes), i.e. at least Gb to Tb.

Computation of W **takes a lot of time, i.e. calculation of** 10^8 **to** 10^{12} **elements.**

Propagation of W **takes a lot of time for calculating** 10^8 **to** 10^{12} **4d integrals.**

The cross spectral density function W can be represented in **coherent modes**:

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n^{\infty} \lambda_n(\omega) \phi_n^*(\boldsymbol{r}_1, \omega) \phi_n(\boldsymbol{r}_2, \omega)$$

Trade **4d** spatial dependencies to **sum of 2d** at fixed frequency.

$\phi_n(\boldsymbol{r}, \boldsymbol{\omega})$ coherent mode

$\lambda_n(\omega)$ eigenvalue (*mode intensities*)

Some coherent mode properties:

- Orthonormal (uncoupled in L_2 sense)
- Fully coherent if and only if one coherent mode
- $d_n(\omega) = \frac{\lambda_n(\omega)}{\sum_j \lambda_j(\omega)}$ occupation (mode distribution or spectrum)
- Maximizing spectral density (compact, controlled)

Each mode propagate like a wavefront so one can build the CDS at any point by propagating the modes to that point.

COMSYL

The coherent modes are the solution of the homogenous Fredholm equation of second kind:

$$A_W[\phi_n] = \lambda_n \phi_n$$

i.e. an eigenvalue problem for:

$$A_W[f](\boldsymbol{r}_2) = \int W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) f(\boldsymbol{r}_1) d\boldsymbol{r}_1$$

Solved by *COMSYL* (Coherent modes for synchrotrons)
Open-source at:

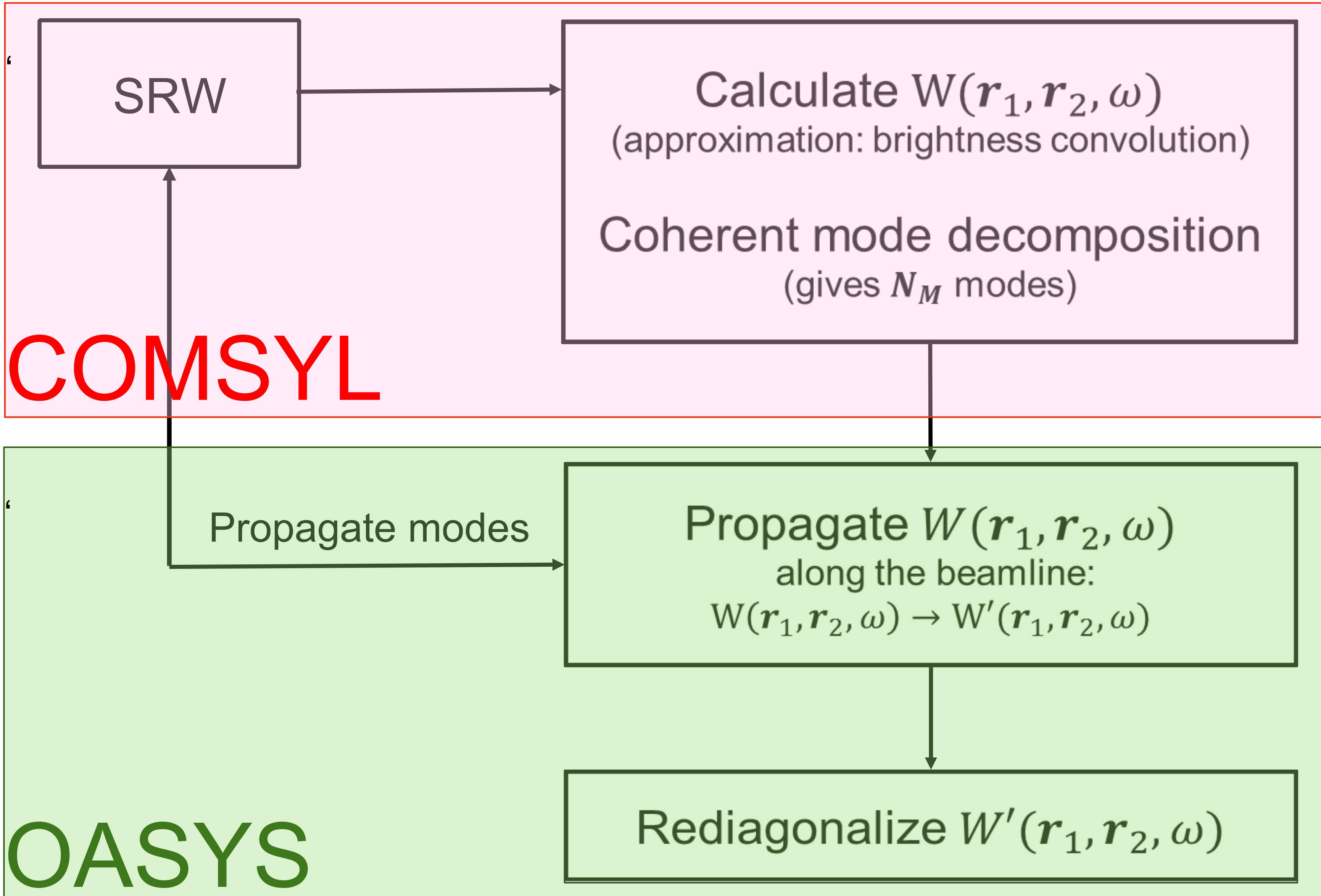
https://github.com/mark-glass/comsyl

Coherent modes of X-ray beams emitted by undulators in new storage rings
Mark Glass and Manuel Sanchez del Rio
EPL, 119 3 (2017) 34004

DOI: https://doi.org/10.1209/0295-5075/119/34004

Free preprint: https://arxiv.org/abs/1706.04393

See the COMSYL Wiki pages https://github.com/mark-glass/comsyl/wiki for more information including the full thesis manuscript.

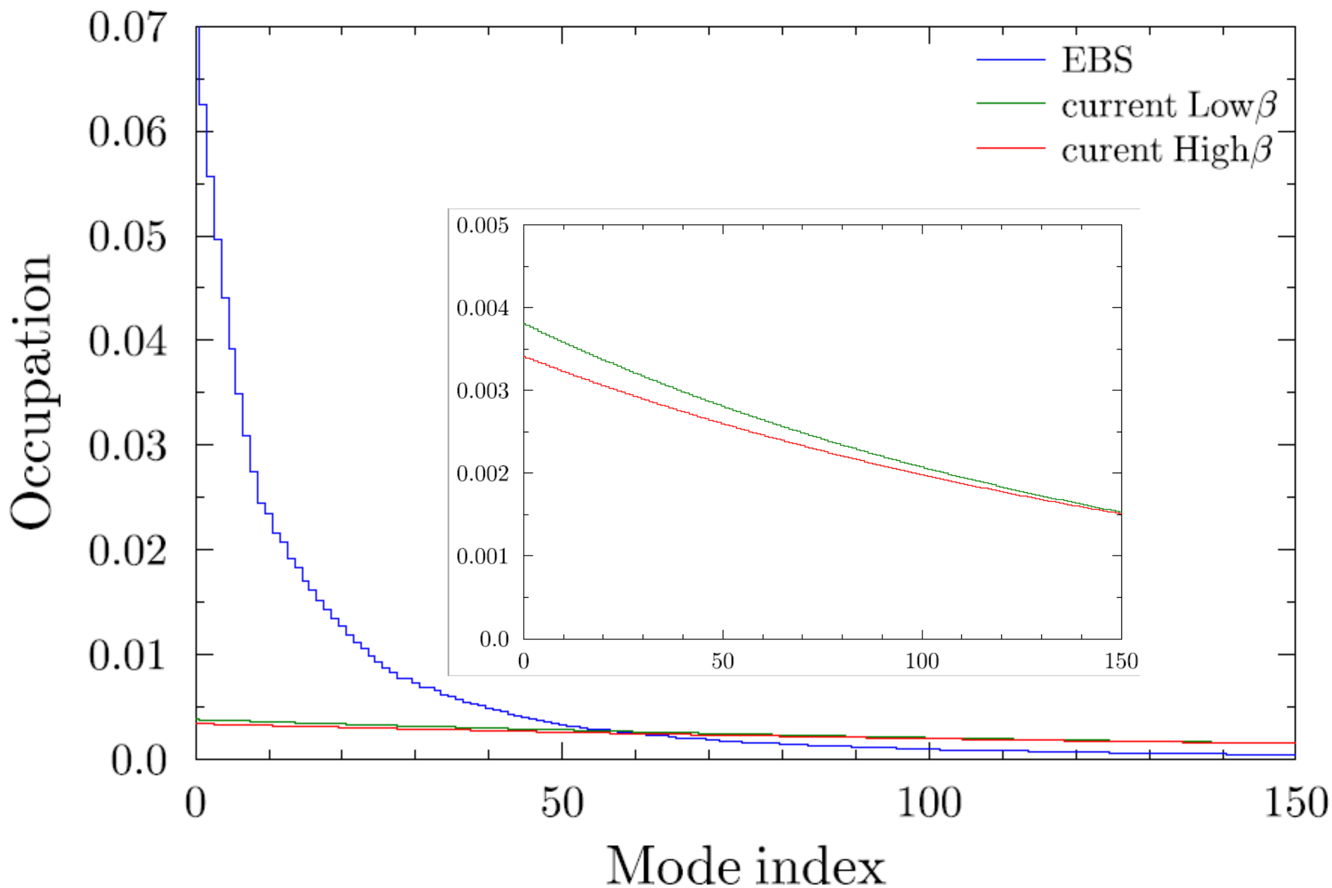

SRW
Calculate $W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
(approximation: brightness convolution)
Coherent mode decomposition
(gives N_M modes)
COMSYL
Propagate modes
Propagate $W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
along the beamline:
$W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) \rightarrow W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
Rediagonalize $W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
OASYS

COMPARISON EBS VS CURRENT LATTICE: MODE SPECTRUM

$\lambda_n(\omega)$ eigenvalue (*mode intensities*)

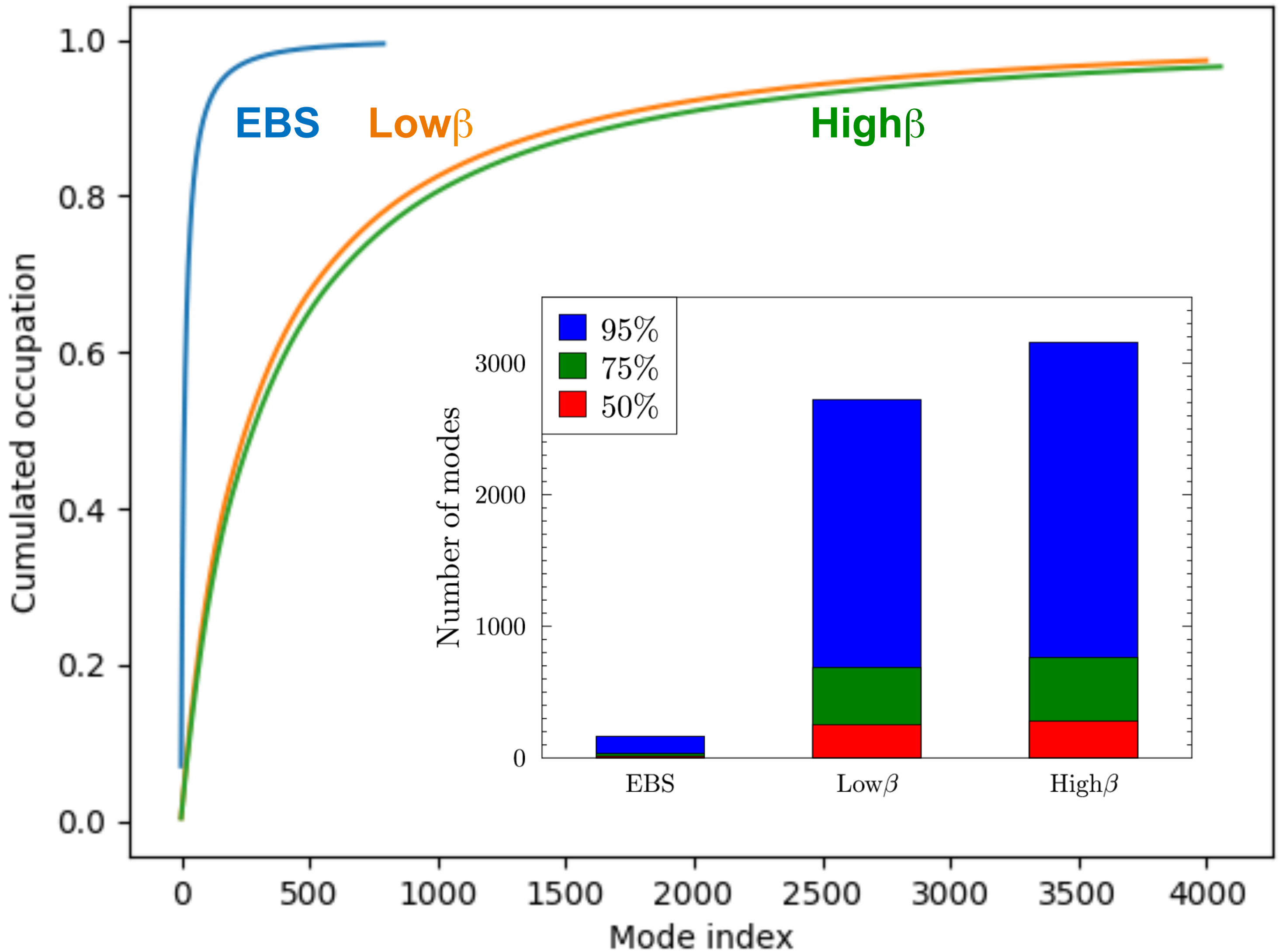
$d_n(\omega) = \frac{\lambda_n(\omega)}{\sum_j \lambda_j(\omega)}$ occupation (mode distribution)

d_0 is the Coherent fraction



2m U18 @ current or EBS, with energy spread, 1.harmonic (8keV)

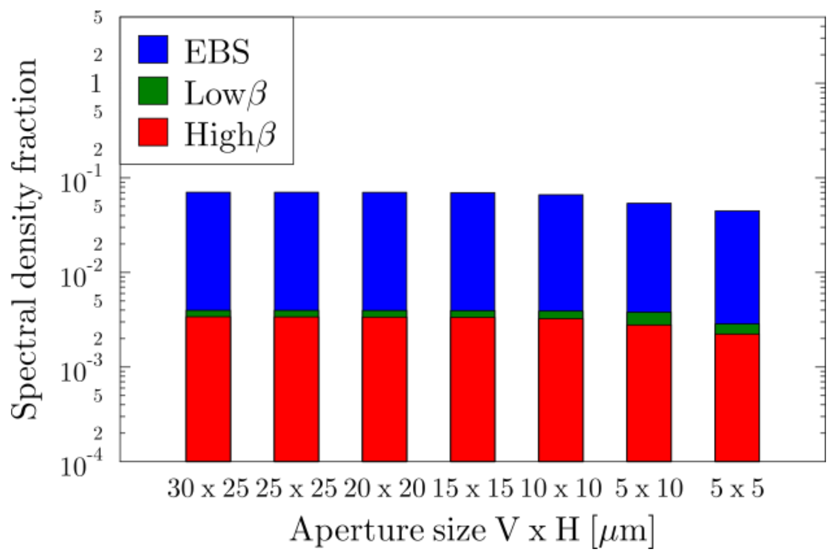
CUMULATED OCCUPATION

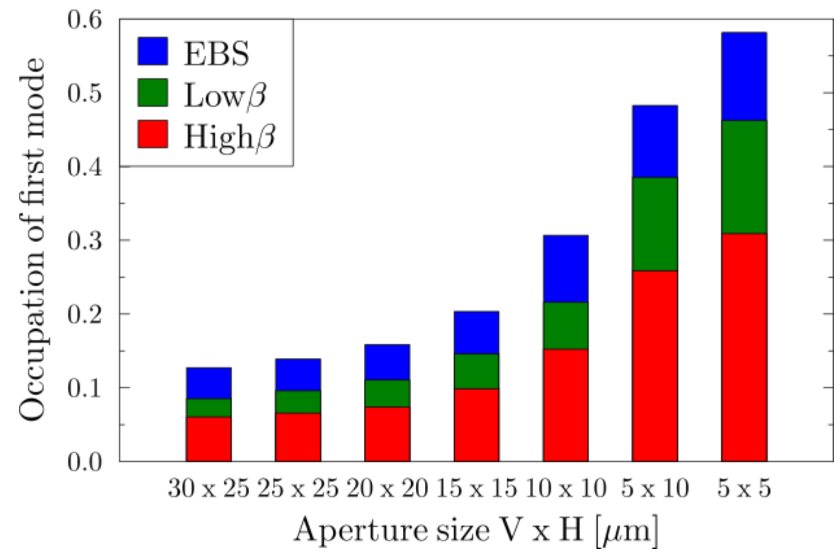


A typical “coherence beamline”: 2m U18 E_0=8keV

U18 2m Eo=8keV   Screen S1 26m   Ideal Lens 36m   Slit S2 2*36m   Screen S3 74m

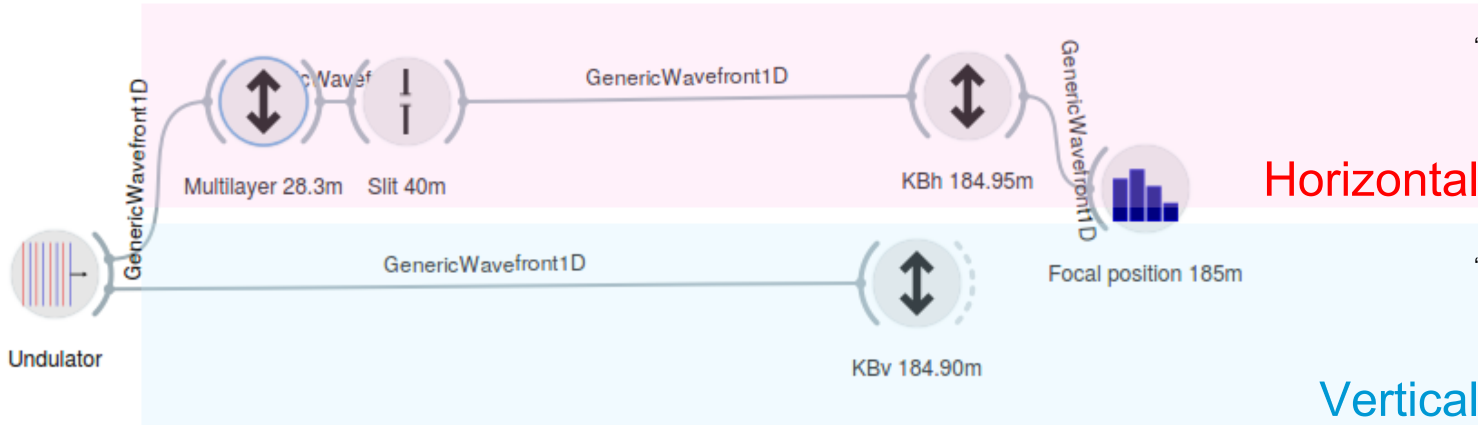
At S_3 after second diagonalization:





Quantifies “less flux” but “more coherent”

EBS will deliver more flux and more coherence

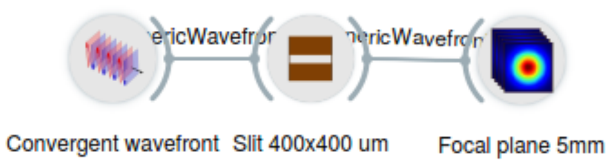


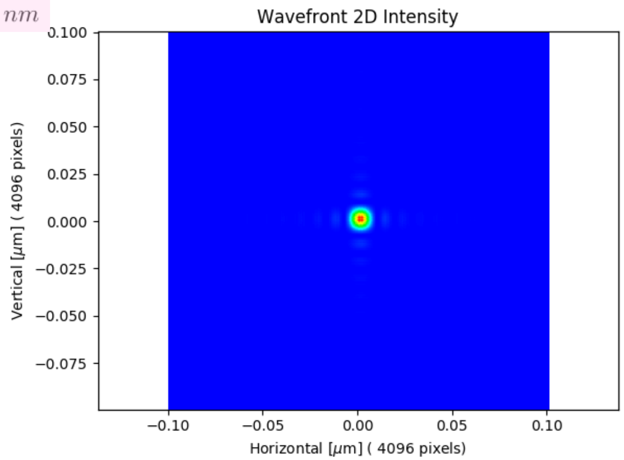
1) Extreme demagnification

Plane	Source	Multilayer	Slit	KB(V)	KB(H)	focal plane
D	0.0	28.3	40.0	184.90	184.95	185.0
H		2.42:1			2899:1	
V				1849:1		

	H	V
ESRF Source	977.2 μm	9.6 μm
ESRF Slit Plane	403.8 μm	
ESRF Focal Plane	139.3 nm	5.2 nm
EBS Source	71.3 μm	10.0 μm
EBS Slit Plane	29.4 μm	
EBS Focal Plane	10.2 nm	5.4 nm

2) Diffraction limited (mirror clipping)

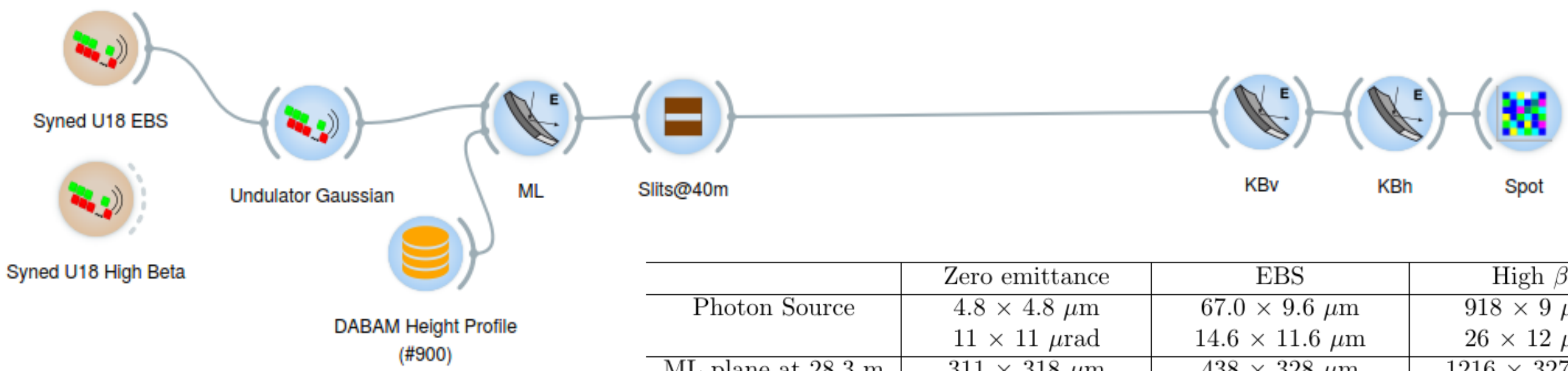




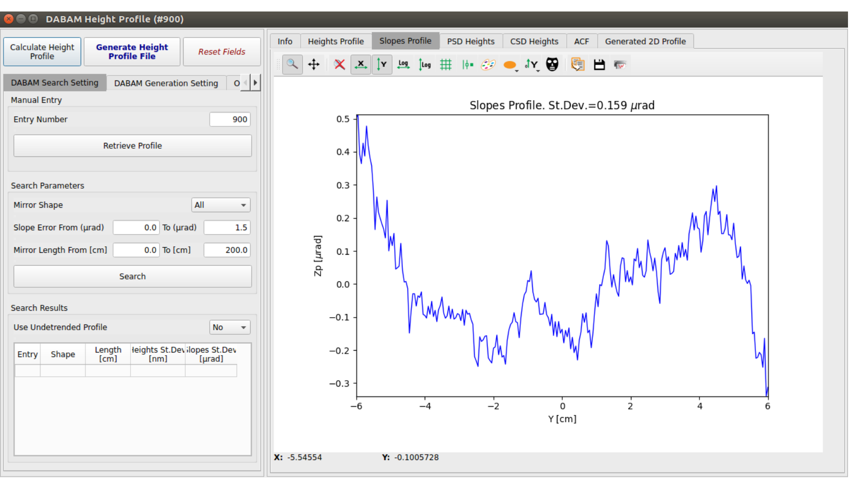
3) Slope errors

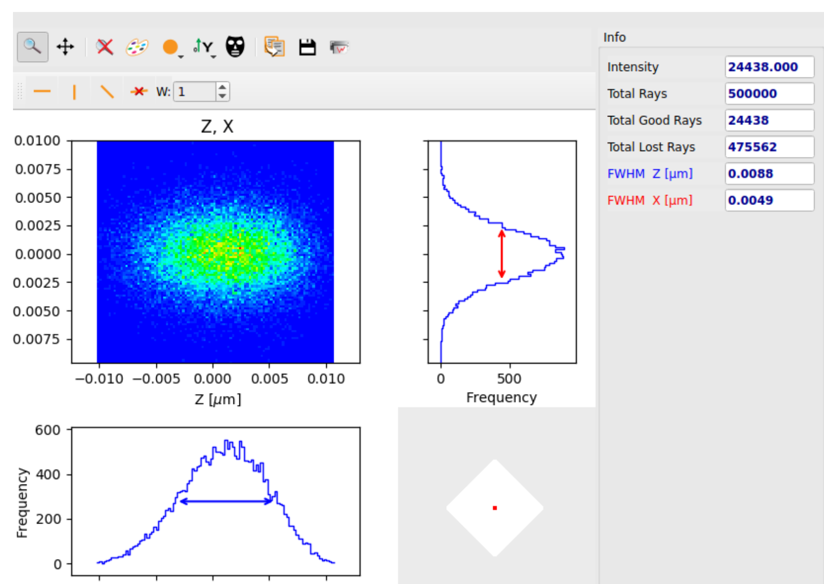
J. C. da Silva et al.: https://doi.org/10.1364/OPTICA.4.000492

RAY TRACING

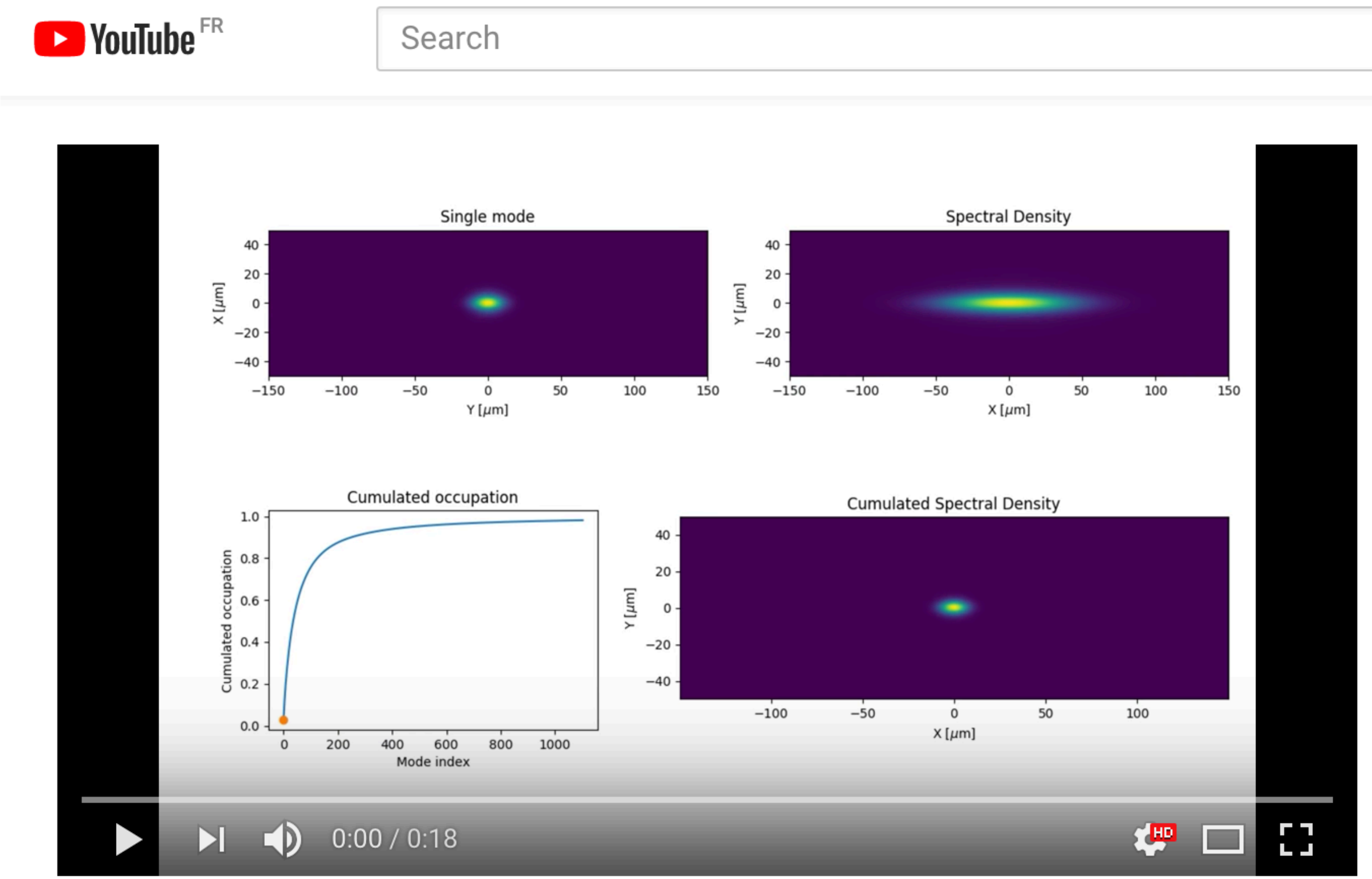


	Zero emittance	EBS	High β
Photon Source	$4.8 \times 4.8\ \mu m$ $11 \times 11\ \mu rad$	$67.0 \times 9.6\ \mu m$ $14.6 \times 11.6\ \mu m$	$918 \times 9\ \mu m$ $26 \times 12\ \mu m$
ML plane at 28.3 m	$311 \times 318\ \mu m$	$438 \times 328\ \mu m$	$1216 \times 327\ \mu m$
Slit at 40 m	$3 \times 431\ \mu m$ (100%)	$28 \times 445\ \mu m$ (95%)	(11%)
KBv	(38%)	(35%)	(4.3%)
KBh	(6.9%)	(4.9%)	(0.44%)
Focal plane at 185m	1×2.4 nm (6.9%)	8.8×4.9 nm (4.8%)	15×5 nm (0.44%)



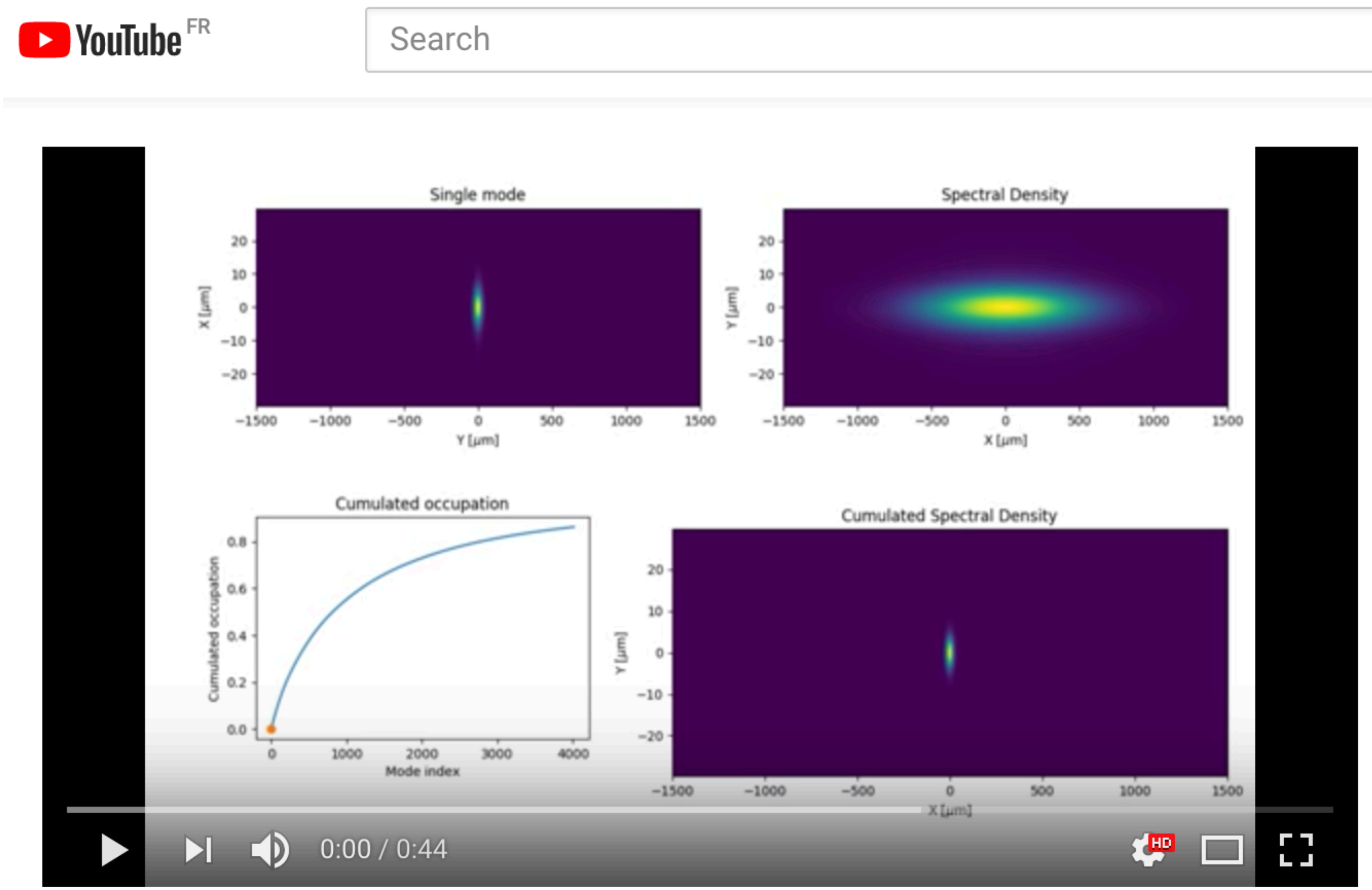


COMSYL EBS SOURCE (CF 0.028)



Coherent modes of synchrotron radiation for EBS

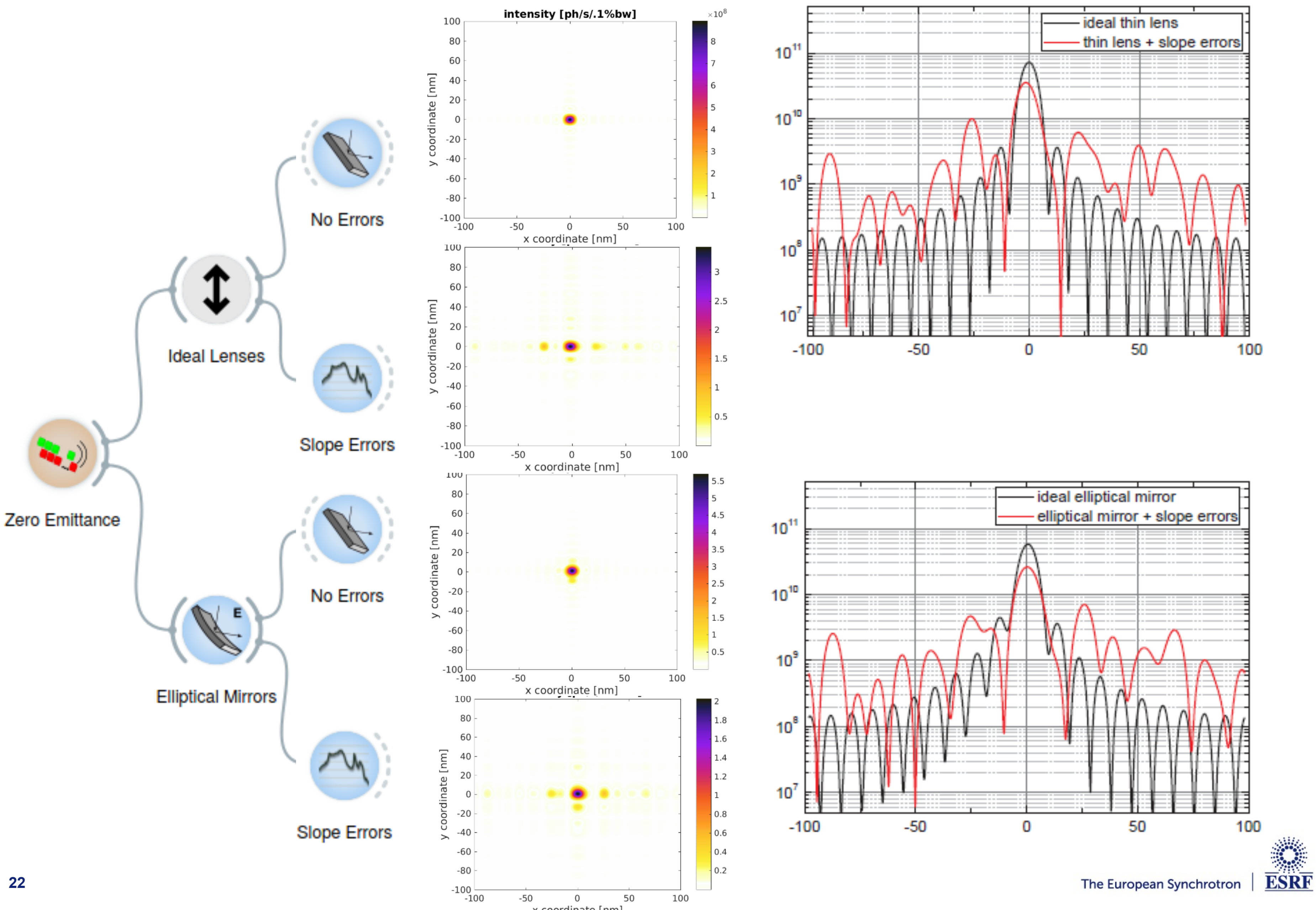
https://youtu.be/h24RrJZaQ80



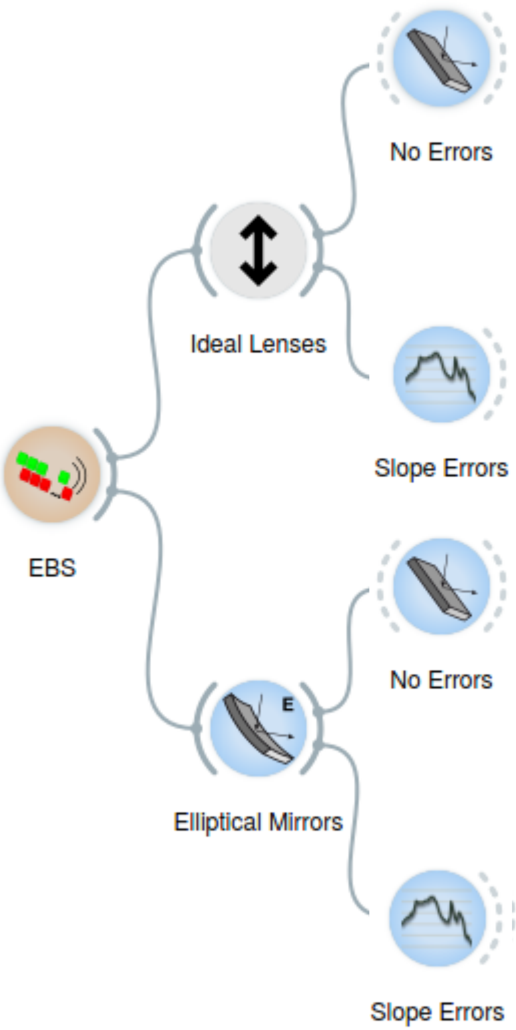
Coherent modes of synchrotron radiation for ESRF-High beta

https://youtu.be/GqJkfv186x0

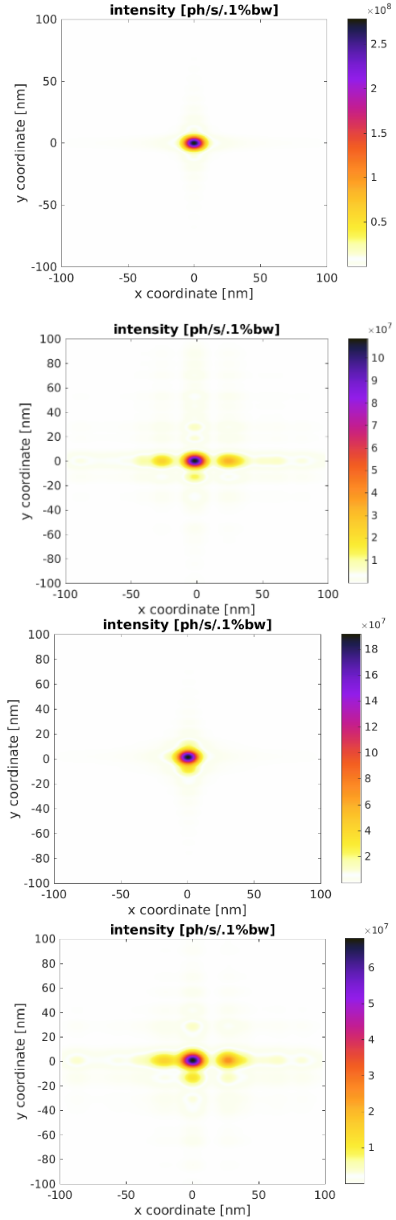
SRW – ZERO EMITTANCE



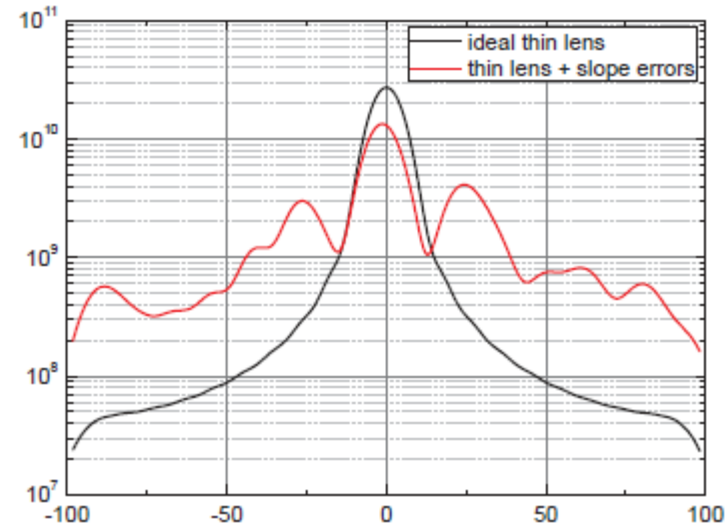
SRW - EBS



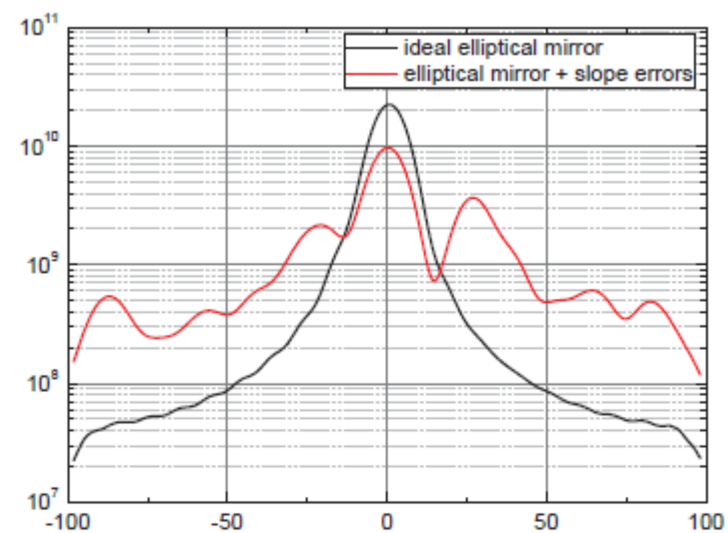
EBS
Ideal Lenses
No Errors
Slope Errors
Elliptical Mirrors
E
No Errors
Slope Errors



intensity [ph/s/.1%bw]
y coordinate [nm]
x coordinate [nm]

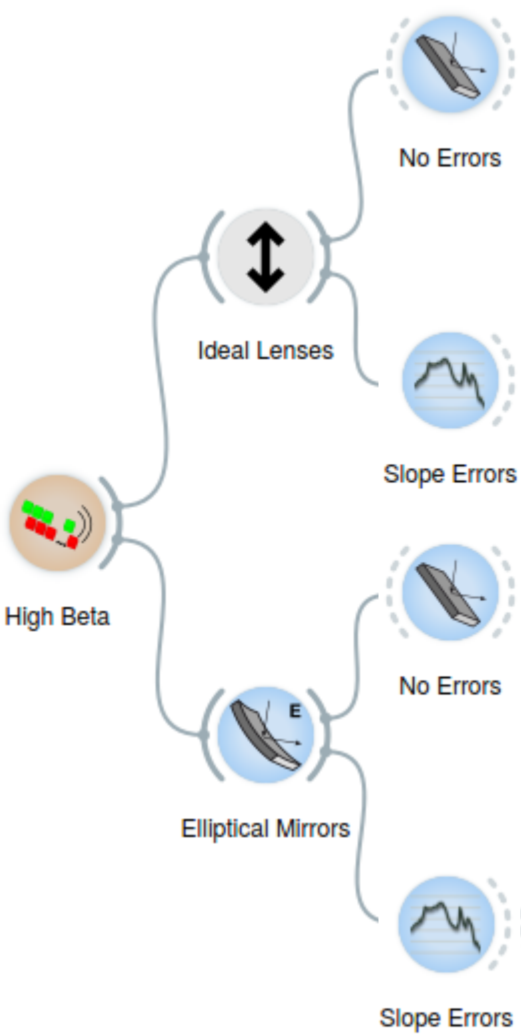


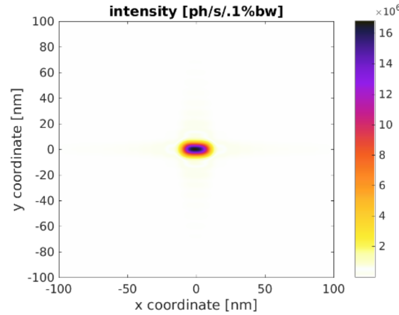
ideal thin lens
thin lens + slope errors

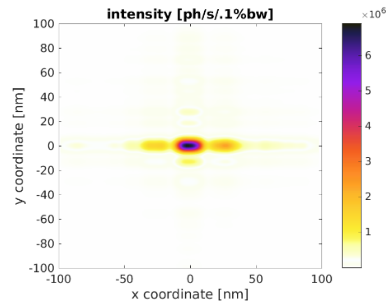


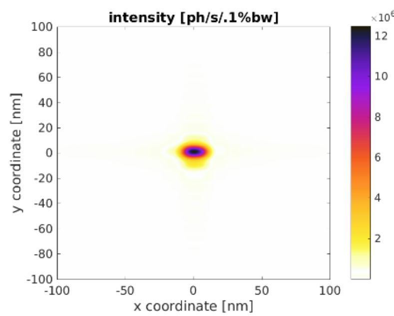
ideal elliptical mirror
elliptical mirror + slope errors

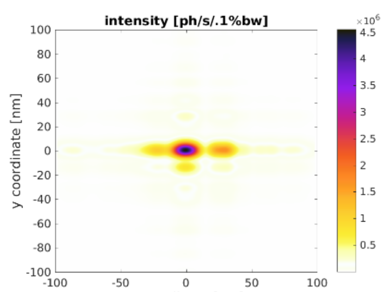
SRW – HIGH BETA

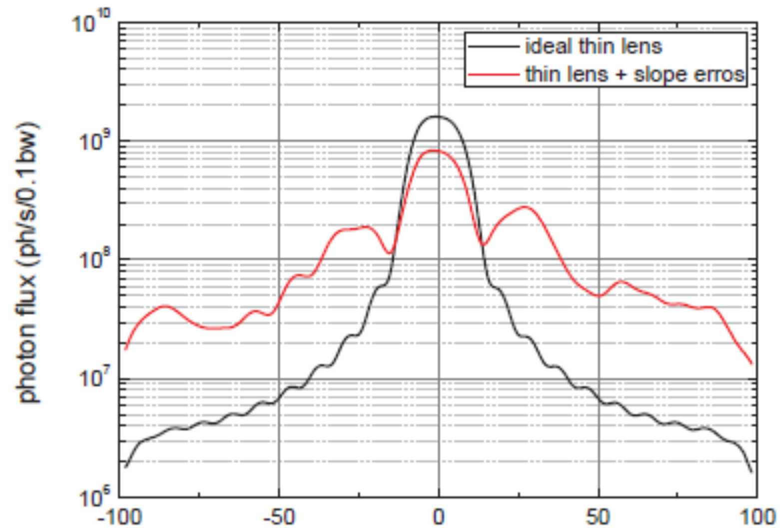


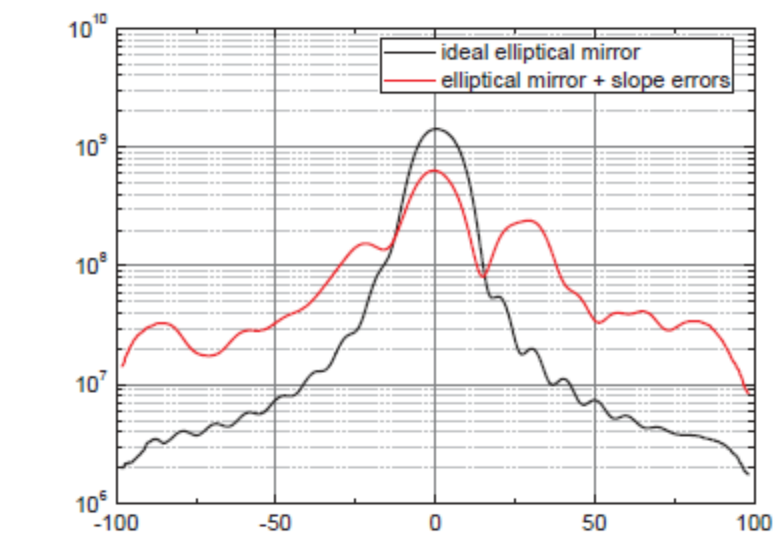












TODO

ID16A

- **Propagate modes**
- **Effect of slope errors:**
  - SHADOW/Hybrid
  - WISE

Complete Oasys Wave Optics Tools

- Communication, propagation and element tools
- 1D simulations to help defining precision parameters
- COMSYL

SUMMARY AND CONCLUSIONS

- **The synchrotron beam emission is due to a collaborative effect of the electrons in a bunch that are responsible of the partial coherence of the beam.**
- **Zero emittance rings are really "diffraction limited" providing a single coherence mode. Upgrade storage ring emission must be treated as partial coherence.**
- **For storage rings emission all coherence properties can be deduced from the Cross Spectral Density. Its storage and propagation is usually unmanagleable by present computers.**
- **COMSYL introduces a new accurate coherent mode decomposition that:**
  - Provides a method of effective storage of CSD
  - Introduces the new concept of "mode spectrum" that quickly summarizes the main coherence properties at a given point of the beamline
  - Computes accurately the coherent fraction
  - Allows to use known propagation methods to propagate modes
  - Permits computing coherent properties of the beam at *any* point of the beamline
- **Applications for a simplified coherence beamline and a nanofocusing ultimate beamline (ID16A) are discussed**

THANKS!

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Julio Cesar Da Silva

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Thank you!